3.5. Financial Markets for Unknown Risks *

1. Introduction

New risks seem to be an unavoidable in a period of rapid change. The last few decades have brought us the risks of global warming, nuclear meltdown, ozone depletion, failure of satellite launcher rockets, collision of supertankers, AIDS and Ebola.[1] A key feature of a new risk, as opposed to an old and familiar one, is that one knows little about it. In particular, one knows little about the chances or the costs of its occurrence. This makes it hard to manage these risks: existing paradigms for the rational management of risks require that we associate probabilities to various levels of losses. This poses particular challenges for the insurance industry, which is at the leading edge of risk management. Misestimation of new risks has lead to several bankruptcies in the insurance and reinsurance businesses.[2] In this paper we propose a novel framework for providing insurance cover against risks whose parameters are unknown. In fact many of the risks at issue may be not just unknown but also unknowable: it is difficult to imagine repetition of the events leading to global warming or ozone depletion, and, therefore, difficult to devise a relative frequency associated with repeated experiments.

A systematic and rational way of hedging unknown risks is proposed here, one which involves the use of securities markets as well as the more traditional insurance techniques. This model is quite consistent with the current evolution of the insurance and reinsurance industries, which are beginning to explore the securitization of some aspects of insurance contracts via Act of God bonds, contingent drawing facilities, catastrophe futures and similar innovations. In fact, our model provides a formal framework within which such moves can be evaluated. An earlier version of this framework was presented in [6]; Chichilnisky [3] gives a more industry-oriented analysis.

This merging of insurance and securities market is not surprising: traditionally economists have recognized two ways of managing risks. One is risk

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pooling, or insurance, invoking the law of large numbers for independent and identically distributed (IID) events to ensure that the insurer's loss rate is proportional to the population loss rate. This will not work if the population loss rate is unknown. The second approach is the use of securities markets, and of negatively correlated events. This does not require knowledge of the population loss rate, and so can be applied to risks which are unknown or not independent. In fact, securities markets alone could provide a mechanism for hedging unknown risks by the appropriate definition of states, but as we shall see below this approach requires an unreasonable proliferation of markets. Using a mix of the two approaches can economize greatly on the number of markets needed and on the complexity of the institutional framework. In the process of showing this, we also show that under certain conditions the market equilibrium is anonymous in the sense that it depends only on the distribution of individuals across possible states, and not on who is in which state.

The reason for using two types of instrument is simple. Agents face two types of uncertainty: uncertainty about the overall incidence of a peril, i.e., how many people overall will be affected by a disease, and then given an overall distribution of the peril, they face uncertainty about whether they will be one of those who are affected. Securities contingent on the distribution of the peril hedge the former type of uncertainty: contingent insurance contracts hedge the latter.

Our analysis implies that insurance companies should issue insurance contracts which depend on the frequency of the peril, which we call a statistical state. The insurance companies should offer individuals an array of insurance contracts, one valid in each possible statistical state. Insurance contracts are, therefore, contingent on statistical states. Within each statistical state, of course, probabilities are known. Therefore, companies are writing insurance only on known risks, something which is actuarially manageable. Individuals then buy the insurance that they want between different statistical states via the markets for securities that are contingent on statistical states. The following is an illustration for purchasing insurance against AIDS, if the actuarial risks of the disease are unknown. One would buy insurance against AIDS by (1) purchasing a set of AIDS insurance contracts each of which pays off only for a specified incidence of AIDS in the population as a whole, and (2) making bets via statistical securities on the incidence of AIDS in the population. Likewise, one would obtain cover against an effect of climate change by (1) buying insurance policies specific to the risks faced at particular levels of climate change, and (2) making bets on the level of climate change, again using statistical securities. The opportunity to place such bets is currently provided in a limited way by catastrophe futures markets which pay an amount depending on the incidence of hurricane damage.

The present paper draws on recent findings of Chichilnisky and Wu [5] and Cass et al. [4], both of which study resource allocation with individual risks.

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Both of these papers develop further Malinvaud's [15, 16] original formulation of general equilibrium with individual risks, and Arrow's [1] formulation of the role of securities in the optimal allocation of risk-bearing. Our results are valid for large but finite economies with agents who face unknown risks and who have diverse opinions about these risks: in contrast, Malinvaud's results are asymptotic, valid for a limiting economy with an infinite population, and deal only with a known distribution of risks. Our results use the formulation of incomplete asset markets for individual risks used to study default in [5, section 5.c]. The risks considered here are unknown and possibly unknowable, and each individual has potentially a different opinion about these risks, while Chichilnisky and Wu [5] and Cass et al. [4] assume that all risk is known.

2. Notation and Definitions

Denote the set of possible states for an individual by S, indexed by $s = 1, 2, \ldots, S$. Let there be H individuals, indexed by $h = 1, 2, \ldots, H$. All households have the same state-dependent endowments: endowments depend solely on the household's individual state s, and this dependence is the same for all households. The probability of any agent being in any state is unknown, and the distribution of states over the population as a whole is also unknown. A complete description of the state of the economy, called a *social state*, is a list of the states of each agent. There are S^H possible social states. A social state is denoted σ : it is an H-vector. The set of possible social states is denoted Ω and has S^H elements. A statistical description of the economy, called a *statistical state*, is a statement of the fraction of the population in each state: it is an S-vector. There are $\binom{H+S-1}{S-1}$ statistical states. Clearly many social states map into a given statistical state. For example, if in one social state you are well and I am sick and in another, I am well and you are sick, then these two social states give rise to the same statistical state. Intuitively, we would not expect the equilibrium prices of the economy to differ in these two social states. One of our results shows that under certain conditions, the characteristics of the equilibrium are in fact dependent only on the statistical state.

How does the distinction between social and statistical states contribute to risk management? Using the traditional approach, we could in principle trade securities contingent on each of the S^H social states. Clearly this would require a large number of markets, a number which grows rapidly with the number of agents. The institutional requirements can be greatly simplified. When the characteristics of the equilibrium depend only on the statistical state, one can trade securities which are contingent on statistical states, i.e., contingent on the distribution of individual states within the population, and still attain efficient allocations. We will trade securities contingent on whether

4 or 8% of the population are in state 5, but not on which people are in this state. Such securities, which we will call *statistical securities*, plus mutual insurance contracts also contingent on the statistical state, lead (under the appropriate conditions) to an efficient allocation of risks. A mutual insurance contract contingent on a statistical state pays an individual a certain amount in a given individual state if and only if the economy as a whole is in a given statistical state.

Let $z_{jh\sigma}$ denote the quantity of good j consumed by household h in social state σ : $z_{h\sigma}$ is an N-dimensional vector of *all* goods consumed by h in social state σ, $z_{h\sigma} = z_{jh\sigma}$, $j = 1, \ldots, N$ and z_h is an NS^H-dimensional vector of *all goods consumed in all social states* by h, $z_h = z_{h\sigma}, \sigma \in \Omega.$[3]

Let $s(h, \sigma)$ be the state of individual h in the social state σ, and $r_s(\sigma)$ be the proportion of all households for whom $s(h, \sigma) = s$. Let $r(\sigma) = r_1(\sigma), \ldots, r_S(\sigma)$ be the distribution of households among individual states within the social state σ, i.e., the proportion of all individuals in state s for each s. $r(\sigma)$ is a statistical state. Let R be the set statistical states, i.e., of vectors $r(\sigma)$ when σ runs over Ω. R is contained in S^I, the product of I S-dimensional simplices, and has $\binom{H+S-1}{S-1}$ elements.

Π^h is household h's probability distribution over the set of social states Ω, and Π^h_σ denotes the probability of state σ. Although we take social states as the primitive concept, we in fact work largely with statistical states. We, therefore, relate preferences, beliefs and endowments to statistical states. This is done in the next section: clearly any distribution over social states implies a distribution over statistical states.

The following *anonymity assumption* is required:

$$r(\sigma) = r(\sigma') \to \Pi^h_\sigma = \Pi^h_{\sigma'}.$$

This means that two overall distributions σ and σ' which have the same statistical characteristics are equally likely. Then Π^h_σ defines a probability distribution Π^h_r on the space of statistical states R. Π^h_r can be interpreted, as remarked above, as h's distribution over possible distributions of impacts in the population as a whole. The probability that a statistical state r obtains and that simultaneously, for a given household h a particular state s also obtains, Π^h_{sr}, is[4]

$$\Pi^h_{sr} = \Pi^h_r r_s \quad \text{with} \quad \sum_s \Pi^h_{sr} = \Pi^h_r. \tag{1}$$

The probability Π^h_s that, for a given h, a particular individual state s obtains is, therefore, given by

$$\Pi^h_s = \sum_{r \in R} \Pi^h_r r_s,$$

where r_s is the proportion of people in individual state s in statistical state r. Note that we denote by $\Pi^h_{s|r}$ the conditional probability of household h being

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$$\Pi^h_{s|r} = r_s,$$

i.e., that the probability of anyone being in individual state s contingent on the economy being in statistical state r is the relative frequency of state s contingent on statistical state r.

3. The Behavior of Households

Let e^h_s be the endowment of household h when the individual state is s. We assume that household h always has the same endowment in the individual state s, whatever the social state. We also assume that all households have the same endowment if they are in the same individual state: endowments differ, therefore, only because of differences in individual states. This describes the risks faced by individuals.

Individuals have von Neumann–Morgenstern utilities:

$$W^h(z_h) = \sum_\sigma \Pi^h_\sigma U^h(z_{h\sigma}).$$

This definition indicates that household h has preferences on consumption which may be represented by a "state separated" utility function W^h defined from elementary state-dependent utility functions.

We assume like Malinvaud [15] that *preferences are separable over statistical states*. This means that the utility of household h depends on σ only through the statistical state $r(\sigma)$. If we assume further that in state σ household h takes into account only its individual consumption, and what overall frequency distribution $r(\sigma)$ appears, and nothing else, then its consumption plan can be expressed as $z^h_\sigma = z_{hsr}$: its consumption depends only on its individual state s and the statistical state r. Summation with respect to social states σ in the expected utility function can now be made first within each statistical state. Hence we can express individuals' utility functions as:

$$W^h(z_{h\sigma}) = \sum_{r,s} \Pi^h_{sr} U^h(z_{hsr}), \tag{2}$$

which expresses the utility of a household in terms of its consumption at individual state s within a statistical state r, summed over statistical states. This expression is important in the following results, because it allows us to represent the utility of consumption across social states σ as a function of statistical states r and individual states s only. The functions U^h_s are assumed to be C^2, strictly increasing, strictly quasiconcave, and the closure of the indifference surfaces $\{U^h_s\}^{-1}(x) \subset \text{int}(R^{N+})$ for all $x \in R^+$. The probabilities Π^h_σ are in principle different over households.

4. Efficient Allocations

Let p^* be a competitive equilibrium price vector of the Arrow–Debreu economy E with markets contingent on all social states[5] and let z^* be the associated allocation. We will as usual say that z^* is *Pareto efficient* if it is impossible to find an alternative feasible allocation which is preferred by at least one agent and to which no agent prefers z^*. Let p_σ^* and z_σ^* be the components of p^* and z^*, respectively, which refer to goods contingent on state σ.

We now define an Arrow–Debreu economy E, where markets exists contingent on an exhaustive description of all states in the economy, i.e. for all social states $\sigma \in \Omega$. We, therefore, have NS^H contingent markets. An *Arrow–Debreu equilibrium* is a price vector $p^* = (p_\sigma), p_\sigma \in R^{N+}, \sigma \in \Omega$, and an allocation z^* consisting of vectors $z_h^* = \left(z_{h\sigma}^*\right), z_{h\sigma}^* \in R^{N+}, \sigma \in \Omega, h = 1, \ldots, H$ such that for all h, z_h^* maximizes

$$W^h\left(z_h^*\right) = \sum_\sigma \Pi_\sigma^h U^h\left(z_{h\sigma}^*\right) \tag{3}$$

subject to a budget constraint

$$p\left(z_h^* - e_h\right) = 0 \tag{4}$$

and all markets clear:

$$\sum_h \left(z_h^* - e_h\right) = 0. \tag{5}$$

Proposition 1 considers the case when households agree on the probability distribution over social states, this common probability being denoted by Π. It follows that they agree on the distribution over statistical states. It shows that in this case, the competitive equilibrium prices p^* and allocations z^* are the same across all social states σ leading to the same statistical state r.[6]

PROPOSITION 1. *When agents have common probabilities, i.e., $\Pi^h = \Pi^j$ $\forall h, j$, then equilibrium prices depend only on statistical states. Consider an Arrow–Debreu equilibrium of the economy E, $p^* = (p_\sigma^*)$, $z^* = (z_\sigma^*)$, $\sigma \in \Omega$. For every state σ leading to a given statistical state r, i.e., such that $r(\sigma) = r$, equilibrium prices and consumption allocations are the same, i.e., there exists a price vector p_r^* and an allocation z_r^* such that $\forall \sigma : r(\sigma) = r$, $p_\sigma^* = p_r^*$ and $z_\sigma^* = z_r^*$, where $p_r^* \in R^{N+}$ and $z_r^* \in R^{NI}$ depend solely on r.*
    Proof. In the Appendix.

DEFINITION. An economy E is *regular* if at all equilibrium prices in E the Jacobian matrix of first partial derivatives of its excess demand function has full rank [11]. Regularity is a generic property [10, 11].

We now consider the general case, which allows for $\Pi^h \neq \Pi^j$ if $h \neq j$. Proposition 1 no longer holds: the reason is that households may not achieve

full insurance at an equilibrium. However, Proposition 2 states that if the economy is *regular, if all households have the same preferences* and if there are two individual states, there is always one equilibrium at which prices are the same at all social states leading to the same statistical state. This confirms the intuition that the characteristics of an equilibrium should not be changed by a permutation of individuals: if I am changed to your state, and you to mine, everyone else remaining constant, then provided you and I have the same preferences, the equilibrium will not change.

PROPOSITION 2. *An Arrow–Debreu equilibrium allocation of the economy $E(p^*, z^*)$ is not fully insured if $\Pi^h \neq \Pi^k$ for some households h, k with $U^h \neq U^k$ in (2). In particular, household h has a different equilibrium allocation across social states σ_1 and σ_2 with $r(\sigma_1) = r(\sigma_2)$. When E is a regular economy, all agents have the same utilities,[7] and there are two individual states, then one of the equilibrium prices p^* must satisfy $p^*_{\sigma_1} = p^*_{\sigma_2}$ for all σ_1, σ_2 with $r(\sigma_1) = r(\sigma_2)$.*

Proof. In the Appendix.

5. Equilibrium in Incomplete Markets for Unknown Risks

Consider first the case where *there are no assets to hedge against risk*, so that the economy has incomplete asset markets. Individuals cannot transfer income to the unfavorable states. Examples are cases when individuals are not able to purchase hurricane insurance, as in some parts of the south eastern United States and in the Caribbean. Market allocations are typically inefficient in this case, since individuals cannot transfer income from one state to another to equalize welfare across states. Which households will be in each individual state is unknown. Each individual has a certain probability distribution over all possible social states σ, Π^h. In each social state σ each individual is constrained in the value of her/his expenditures by her/his endowment (which depends on the individual state $s(h, \sigma)$ in that social state). In this context, a *general equilibrium of the economy with incomplete markets E_I* consists of a price vector p^* with NS^H components and H consumption plans z^*_h with NS^H components each, such that z^*_h maximizes $W^h(z_h)$:

$$W^h(z_h) = \sum_{\sigma} \Pi^h_{\sigma} U^h(z_{h\sigma}) \tag{6}$$

subject to

$$p_{\sigma}(z_{h\sigma} - e_{h\sigma}) = 0 \quad \text{for each} \quad \sigma \in \Omega \tag{7}$$

and

$$\sum_{h=1}^{H}(z_h - e_h) = 0. \tag{8}$$

The above economy E_I is an extreme version of an economy with incomplete asset markets (see, e.g., [13]) because there are no markets to hedge against risks: there are S^H budget constraints in (7).

6. Efficient Allocations, Mutual Insurance and Securities

In this section we study the possibility of supporting Arrow–Debreu equilibria by combinations of statistical securities and insurance contracts, rather than by using state contingent contracts. As already observed, this leads to a very significant economy in the number of markets needed. In an economy with no asset markets at all, such as E_I, the difficulty in supporting an Arrow–Debreu equilibrium arises because income cannot be transferred between states. On the basis of Propositions 1 and 2, we show that households can use securities defined on statistical states to transfer into each such state an amount of income equal to the expected difference between the value of Arrow–Debreu equilibrium consumption and the value of endowments in that state. The expectation here is over individual states conditional on being in a given statistical state. The difference between the actual consumption-income gap given a particular individual state and its expected value is then covered by insurance contracts. Recall that A is the binomial number $A = \binom{H+S-1}{S-1}$.

THEOREM 1. *Assume that all households in E have the same probability Π over the distribution of risks in the population. Then any Arrow–Debreu equilibrium allocation (p^*, z^*) of E (and, therefore, any Pareto Optimum) can be achieved within the general equilibrium economy with incomplete markets E_I by introducing a total of $I.A$ mutual insurance contracts to hedge against individual risk, and A statistical securities to hedge against social risk. In a regular economy with two individual states and identical preferences, even if agents have different probabilities, there is always an Arrow–Debreu equilibrium (p^*, z^*) in E which is achievable within the incomplete economy E_I with the introduction of $I.A$ mutual insurance contracts and A statistical securities.*
    Proof. In the Appendix.

6.1. *Market Complexity*

We can now formalize a statement made before about the efficiency of the institutional structure proposed in Theorem 1 by comparison with the standard Arrow–Debreu structure of a complete set of state-contingent markets. We use here complexity theory, and in particular the concept of *NP-completeness*. The key consideration in this approach to studying problem complexity is how fast the number of operations required to solve a problem increases with the size of the problem.

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DEFINITION. If the number of operations required to solve a problem must increase exponentially for any possible way of solving the problem, then the problem is called *"intractable"* or more formally, *NP-complete*. If this number increases polynomially, the problem is *tractable*. Further definitions are in [12].

The motivation for this distinction is of course that if the number of operations needed to solve the problem increases exponentially with some measure of the size of the problem, then there will be examples of the problem that no computer can or ever could solve. Hence there is no possibility of ever designing a general efficient algorithm for solving these problems. However, if the number of operations rises only polynomially then it is in principle possible to devise a general and efficient algorithm for the problem.

Theorem 2 investigates the complexity of the resource allocation problem in the Arrow–Debreu framework and compares this with the framework of Theorem 1. We focus on how the problem changes as the economy grows in the sense that the number of households increases, and consider a very simple aspect of the allocation problem, which is as follows. Suppose that the excess demand of the economy $Z(p)$ is known. A particular price vector p^* is proposed as a market clearing price. We wish to check whether or not it is a market clearing price. This involves computing each of the coordinates of $Z(p)$ and then comparing with zero. This involves a number of operations proportional to the number of components of $Z(p)$; we, therefore, take the rate at which the dimension of $Z(p)$ increases with the number of agents to be a measure of the complexity of the resource allocation problem. In summary: we ask how the difficulty of verifying market clearing increases as the number of households in the economy rises. We show that in the Arrow–Debreu framework this difficulty rises exponentially, whereas in the framework of Theorem 1 it rises only polynomially.

THEOREM 2. *Verifying market clearing is an intractable problem in an Arrow–Debreu economy, i.e., the number of operations required to check if a proposed price is market clearing increases exponentially with the number of households H. However, under the assumptions of Theorem 1, in the economy E_I supplemented by I.A mutual insurance contracts and A statistical securities, verifying market clearing is a tractable problem, i.e., the number of operations needed to check for market clearing increases only polynomially with the number of households.*

Proof. The number of operations required to check that a price is market clearing is proportional to the number of market clearing conditions. In E we have NS^H markets. Hence the number of operations needed to check if a proposed price is market clearing must rise exponentially with the number of households H. Consider now the case of E_I supplemented by $I.A$ mutual insurance contracts and A securities. Under the assumptions of Theorem 1,

by Propositions 1 and 2, we need only check for market clearing in one social state associated with any statistical state, as if markets clear in one social state leading to a certain statistical state they will clear in all social states leading to the same statistical state. Hence we need to check a number of goods markets equal to $N.A$, plus markets for mutual insurance contracts and securities. Now

$$A = \binom{H + S + 1}{S - 1} = \Phi(H, S),$$

where $\Phi(H, S)$ is a polynomial in H of order $(S - 1)$. Hence A itself is a polynomial in H whose highest order term depends on H^{S-1}, completing the proof. □

7. Catastrophe Futures and Bundles

We mentioned in the introduction that securities contingent on statistical states are already traded as "catastrophe futures" on the Chicago Board of Trade, where they were introduced in 1994. Recently, hurricane bonds and earthquake bonds have been introduced, additional examples of statistical securities. (The concept was discussed by Chichilnisky and Heal in 1993 [6].) Catastrophe futures are securities which pay an amount that depends on the value of an index of insurance claims paid during a year. One such index measures the value of hurricane damage claims: others measure claims stemming from different types of natural disasters. The value of hurricane damage claims depends on the overall incidence of hurricane damage in the population, but is not of course affected by whether any particular individual is harmed. It, therefore, depends, in our terminology, on the statistical state, on the distribution of damage in the population, but not on the social state. Catastrophe futures are thus financial instruments whose payoffs are conditional on statistical state of the economy: they are statistical securities. According to our theory, a summary version of which appeared in [6] in 1993, they are a crucial prerequisite to the efficient allocation of unknown risks. And as the incidence and extent of natural disaster claims in the U.S. has increased greatly in recent years, risks such as hurricane risks are in effect unknown risks: insurers are concerned that the incidence of storms may be related to trends in the composition of the atmosphere and incipient greenhouse warming. However, catastrophe futures are not on their own sufficient for this: they do not complete the market. Mutual insurance contracts, as described above, are also needed. These provide insurance conditional on the value of the catastrophe index. The two can be combined into "catastrophe bundles", see [3].

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8. Conclusions

We have defined an economy with unknown individual risks, and established that a combination of statistical securities and mutual insurance contracts can be used to obtain an efficient allocation of risk-bearing. Furthermore, we have shown that this institutional structure is efficient in the sense that it requires exponentially fewer markets that the standard approach via state-contingent commodities. In fact, the state-contingent problem is "intractable" with individual risks (formally, NP-complete) in the language of computational complexity, whereas our approach gives a formulation that is polynomially complex. This greatly increases the economy's ability to achieve efficient allocations. Another interesting feature of this institutional structure is the interplay of insurance and securities markets involved. Its simplicity leads to successful hedging of unknown risks and predicts some convergence between the insurance and securities industries.

9. Appendix

PROPOSITION 1. *When agents have common probabilities, i.e., $\Pi^h = \Pi^j$ $\forall h, j$, then equilibrium prices depend only on statistical states. Consider an Arrow–Debreu equilibrium of the economy E, $p^* = (p_\sigma^*)$, $z^* = (z_\sigma^*)$, $\sigma \in \Omega$. For every state σ leading to a given statistical state r, i.e., such that $r(\sigma) = r$, equilibrium prices and consumption allocations are the same, i.e., there exists a price vector p_r^* and an allocation z_r^* such that $\forall \sigma : r(\sigma) = r$, $p_\sigma^* = p_r^*$ and $z_\sigma^* = z_r^*$, where $p_r^* \in R^{N+}$ and $z_r^* \in R^{NI}$ depend solely on r.*

Proof. Consider σ_1 and σ_2 with $r(\sigma_1) = r(\sigma_2) = r$. Note that the total endowments of the economy are the same in σ_1 and σ_2, both equal to $s_r = Hr_s e_{hs}$ (recall that $e_{hs} = e_s$ as endowments depend only on individual states and not on household identities). Also, by the anonymity assumption, $\Pi_{\sigma_1} = \Pi_{\sigma_2} = \Pi_r$, where Π_r is the common probability of any social state in the statistical state r. Let $\Pi_{\sigma|r}$ be the probability of being in social state σ given statistical state r. By the anonymity assumption on probabilities this is just $1/\#\Omega_r$. We now show that for every household h, $z_{h\sigma_1}^* = z_{h\sigma_2}^*$, due to the Pareto efficiency of Arrow–Debreu equilibria. Let $\Omega_r = \{\sigma : r(\sigma) = \sigma\}$. Let $z^* = (z_{h\sigma}^*)$, and assume in contradiction to the proposition that there are σ_1 and $\sigma_2 \in \Omega_r$ such that $z_{h\sigma_1}^* \neq z_{h\sigma_2}^*$ for some h. Define $Ez_{hr} = \sum_{\sigma \in \Omega_r} z_{h\sigma}^* \Pi_{\sigma|r} = (1/\#\Omega_r) \sum_{\sigma \in \Omega_r} z_{h\sigma}^*$. This is the expected value of $(z_{h\sigma}^*)$ given that the economy is in the statistical state r. Now

$$\sum_h Ez_{hr} = \sum_h \frac{1}{\#\Omega_r} \sum_{\sigma \in \Omega_r} z_{h\sigma}^* = \sum_h z_{h\sigma}^*,$$

so that $Ez_{h\sigma}$ is a feasible consumption vector for each h in the statistical state r. Next we show that by strict concavity, moving for each h and each σ from

$z_{h\sigma}^*$ (which depends on σ) to Ez_{hr} (which is the same for all $\sigma \in \Omega$), is a strict Pareto improvement. This is because

$$W^h\left(z_{h\sigma}^*\right) = \sum_\sigma \Pi_\sigma U^h\left(z_{h\sigma}^*\right) = \sum_r \Pi_r \sum_{\sigma \in \Omega} \Pi_{\sigma|r} U^h\left(z_{h\sigma}^*\right).$$

By strict concavity of preferences,

$$\sum_r \Pi_r \sum_{\sigma \in \Omega_r} \Pi_{\sigma|r} U^h\left(z_{h\sigma}^*\right)$$

$$< \sum_r \Pi_r \sum_{\sigma \in \Omega_r} U^h\left(\sum_{\sigma \in \Omega} z_{h\sigma}^* \Pi_{\sigma|r}\right) = \sum_r \Pi_r \sum_{\sigma \in \Omega} U^h\left(Ez_{h\sigma}\right).$$

Since $Ez_{h\sigma}$ is Pareto superior to z^* with $z_{h\sigma_1}^* \neq z_{h\sigma_2}^*$, such a z^* cannot be an equilibrium allocation. Hence $z_{h\sigma_1}^* = z_{h\sigma_2}^* = z_{hr}^*$ for all $h = 1, \ldots, H$. Note that this implies that in an equilibrium, household h consumes the same allocation z_{hr}^* across all individual states s in a given statistical state, i.e. it achieves full insurance. Since p^* supports the equilibrium allocation z^*, and $z_{h\sigma_1}^* = z_{h\sigma_2}^*$ it follows that $p_{\sigma_1}^* = p_{\sigma_2}^*$ when $r(\sigma_1) = r(\sigma_2)$, because utilities are assumed to be C^2 and, in particular, to have a unique gradient at each point which, by optimality, must be collinear both with $p_{\sigma_1}^*$ and with $p_{\sigma_2}^*$, i.e. $p_{\sigma_1}^* = p_{\sigma_2}^* = p_r^*$. This implies that at an equilibrium, household h faces the same prices p_r^* at any σ with $r(\sigma) = r$. $\qquad\qquad\square$

PROPOSITION 2. *An Arrow–Debreu equilibrium allocation of the economy $E(p^*, z^*)$ is not fully insured if $\Pi^h \neq \Pi^k$ for some households h, k with $U^h \neq U^k$. In particular, household h has a different equilibrium allocation across social states σ_1 and σ_2 with $r(\sigma_1) = r(\sigma_2)$. When E is a regular economy, all agents have the same utilities,[8] and there are two individual states, one of the equilibrium prices p^* must satisfy $p_{\sigma_1}^* = p_{\sigma_2}^*$ for all σ_1, σ_2 with $r(\sigma_1) = r(\sigma_2)$.*

Proof. Suppose that household h is in fact fully insured so that $z_{h\sigma_1}^* = z_{h\sigma_2}^*$ for all σ^1 and σ_2 with $r(\sigma_1) = r(\sigma_2)$. Household h's consumption levels are $y_{s_1 r}^i$ and $y_{s_2 r}^i$ where $s_1 = s(h, \sigma_1)$ and $s_2 = s(h, \sigma_2)$. By assumption we have $y_{s_1 r}^i = y_{s_2 r}^i$. Now from (2) household h's marginal rate of substitution between consumption in states σ_1 and σ_2 is $\Pi_{s_1|r}^h / \Pi_{s_2|r}^h$. Suppose also that household k, $k \neq h$, is fully insured. Then by the same argument k's marginal rate of substitution between consumption in states σ_1 and σ_2 is $\Pi_{s_1|r}^k / \Pi_{s_2|r}^k$. But if different households have different probability distributions this is a contradiction as both face the same price vector.

Assume now that E is regular, that all agents have the same preferences, and that $S = 2$. Consider two social states σ_1 and σ_2 with $r(\sigma_1) = r(\sigma_2)$, and such that σ_1 differs from σ_2 only on the individual states of the two households h_1 and h_2 which are permuted, i.e., $s(h_1, \sigma_1) = s(h_2, \sigma_2)$ and

ame for all $\sigma \in \Omega$), is a

$$\Pi_{\sigma|r} U^h \left(z_{h\sigma}^*\right).$$

$$\Pi_r \sum_{\sigma \in \Omega} U^h \left(Ez_{h\sigma}\right).$$

$_{h\sigma_2}$, such a z^* cannot be $_{r}$ for all $h = 1,\dots,H$. ld h consumes the same en statistical state, i.e. it brium allocation z^*, and $r(\sigma_2)$, because utilities unique gradient at each ith $p_{\sigma_1}^*$ and with $p_{\sigma_2}^*$, i.e. , household h faces the □

location of the economy ie households h, k with t equilibrium allocation). When E is a regular here are two individual $p_{\sigma_1}^* = p_{\sigma_2}^*$ for all σ_1, σ_2

ured so that $z_{h\sigma_1}^* = z_{h\sigma_2}^*$ h's consumption levels ,σ_2). By assumption we inal rate of substitution $I_{s_2|r}^h$. Suppose also that e argument k's marginal l and σ_2 is $\Pi_{s_1|r}^k/\Pi_{s_2|r}^k$. y distributions this is a

e the same preferences, $_2$ with $r(\sigma_1) = r(\sigma_2)$, idual states of the two $_1,\sigma_1) = s(h_2,\sigma_2)$ and

$s(h_2,\sigma_1) = s(h_1,\sigma_2)$. Assume that there exists an equilibrium price for E, $p^* \in R^{NS^H}$, such that its components in states σ_1 and σ_2 are different, i.e. $p_{\sigma_1}^* \neq p_{\sigma_2}^*$. Define now a new price $p_c^* \in R^{NS^H}$, called a "conjugate" of p^*, which differs from p^* only in its coordinates in states σ_1 and σ_2, which are permuted as follows: $\forall \sigma \neq \sigma_1, \sigma_2, \overline{p}_\sigma^* = p_\sigma^*$, $\overline{p}_{\sigma_1}^* = p_{\sigma_2}^*$, and $\overline{p}_{\sigma_2}^* = p_{\sigma_1}^*$. We shall now show that $\overline{p}^*$ is also an equilibrium price for the economy E. At $\overline{p}^*$, household h_1 has the same endowments and faces the same prices in states σ_1 and σ_2 as it did at states σ_2 and σ_1 respectively at price p^*; at all other states $\sigma \in \Omega$, h_1 faces the same prices and has the same endowments facing p^* and facing $\overline{p}^*$. The same is true of household h_2. Furthermore, h_1 and h_2 have the same utilities and probabilities at σ_1 and σ_2 because $r(\sigma_1) = r(\sigma_2)$ and probabilities are anonymous. Therefore, the excess demand vectors of h_1 in states σ_1 and σ_2 at prices p^* equal the excess demand vectors of h_2 in σ_2 and σ_1 respectively, at prices $\overline{p}^*$, and at all other states $\sigma \in \Omega$ the excess demand vectors of h_1 are the same at prices p^* and $\overline{p}^*$. Reciprocally: the excess demand vectors of h_2 in σ_1 and σ_2 at prices p^* equal the excess demand vectors of h_1 in σ_2 and σ_1 respectively at prices $\overline{p}^*$, and in all other states σ, the excess demand vectors of h_2 are the same as they are with prices p^*. Formally:

$$z_{h_1\sigma_1}\left(\overline{p}^*\right) = z_{h_2\sigma_2}\left(p^*\right), z_{h_1\sigma_2}\left(\overline{p}^*\right) = z_{h_2\sigma_1}\left(p^*\right)$$

$$z_{h_2\sigma_1}\left(\overline{p}^*\right) = z_{h_1\sigma_2}\left(p^*\right), z_{h_2\sigma_2}\left(\overline{p}^*\right) = z_{h_1\sigma_1}\left(p^*\right)$$

and $\forall \sigma \in \Omega, \sigma \neq \sigma_1, \sigma_2$:

$$z_{h_1\sigma}\left(p^*\right) = z_{h_1\sigma}\left(\overline{p}^*\right), z_{h_2\sigma}\left(p^*\right) = z_{h_2\sigma}\left(\overline{p}^*\right).$$

The excess demand vectors of all other households $h \neq h_1, h_2$ are the same for p^* and $\overline{p}^*$. Therefore, at $\overline{p}^*$ the aggregate excess demand vector of the economy is zero, so that $\overline{p}^*$ is an equilibrium. The same argument shows that permuting the two components $p_{\sigma_1}^*, p_{\sigma_2}^*$ of a price p^* at any two social states σ_1, σ_2 leading to the same statistical state $r(\sigma_1)$ leads from an equilibrium price p^* to another equilibrium price $\overline{p}^*$. This is because if two social states σ_1 and σ_2 lead to the same statistical state and there are two individual states s_1 and s_2 then there is a number $k > 0$ such that k households who are in s_1 in σ_1 are in s_2 in σ_2 and another k households who were in s_1 in σ_2 are in s_2 in σ_1, while remaining in the same individual states otherwise. These two sets of k households can be paired. For every pair of households, the above argument applies. Hence it applies to the sum of the demands, so that the new price $\overline{p}^*$ is an equilibrium.

Now consider any regular economy E with a finite number of equilibrium prices denoted $p_1^*,\dots,p_k^*$. We shall show that there exists a $j \leq k$ s.t. p_j^* assigns the same price vector to all social states σ_1, σ_2 with $r(\sigma_1) = r(\sigma_2)$. Start with p_1^*: if p_1^* does not have this property, consider the first two social states σ_1, σ_2 with $r(\sigma_1) = r(\sigma_2)$ and $p_{1\sigma_1}^* \neq p_{1\sigma_2}^*$. Define $\overline{p}_1^*$ as the conjugate of p_1^* constructed by permuting the prices of the social states σ_1 and σ_2. If

$\forall j > 1, p_j^* = \bar{p}_1^*$, then there are two price equilibria, i.e. $k = 2$; however, since the number of price equilibria must be odd,[9] there must exist $p_{j_1}^*$ with $j_1 > 1$, and $p_{j_1}^* \neq \bar{p}_1^*$. Consider now the conjugate of $p_{j_1}^*$ with respect to the first two social states σ_1, σ_2 which correspond to the same statistical state and have different components in $p_{j_1}^*$, and denote this conjugate $\bar{p}_{j_1}^*$. Repeat the procedure until all equilibria are exhausted. In each step of this procedure, two different price equilibria are found. Since the number of equilibria must be odd, it follows that there must exist a $j \leq k$ for which all conjugates of p_j^* equal p_j^*: this is the required equilibrium which assigns the same equilibrium prices $p_{\sigma_1}^* = p_{\sigma_2}^*$ to all σ_1, σ_2 with $r(\sigma_1) = r(\sigma_2)$, completing the proof.   □

THEOREM 1. *Assume that all households in E have the same probability Π over the distribution of risks in the population. Then any Arrow–Debreu equilibrium allocation (p^*, z^*) of E (and, therefore, any Pareto Optimum) can be achieved within the general equilibrium economy with incomplete markets E_I by introducing a total of $I.A$ mutual insurance contracts to hedge against individual risk, and A statistical securities to hedge against social risk. In a regular economy with two individual states and identical preferences, even if agents have different probabilities, there is always an Arrow–Debreu equilibrium (p^*, z^*) in E which is achievable within the incomplete economy E_I with the introduction of $I.A$ mutual insurance contracts and A statistical securities.*

Proof. Consider first the case where all households have the same probabilities, i.e., $\Pi^h = \Pi^j = \Pi$. By Proposition 1, an Arrow–Debreu equilibrium of E has the same prices $p_\sigma^* = p_r^*$ and the same consumption vectors $z_{h\sigma}^* = z_{hr}^*$ for each h, at each social state σ with $r(\sigma) = r$. Define $\Omega(r)$ as the set of social states mapping to a given statistical state r, i.e. $\Omega(r) = \{\sigma \in \Omega : r(\sigma) = r\}$. The budget constraint (4) is

$$p^* (z_h^* - e_h) = \sum_\sigma p_\sigma^* \left(z_{h,\sigma}^* - e_{h\sigma} \right) = \sum_r p_r^* \sum_{\sigma \in \Omega(r)} \left(z_{h\sigma}^* - e_{h\sigma} \right) = 0.$$

Individual endowments depend on individual states and not on social states, so that $e_{h\sigma} = e_{hs(\sigma)} = e_{hs}$; furthermore, by Proposition 1 equilibrium prices depend on r and not on σ, so that for each r the equilibrium consumption vector $z_{h\sigma}$ can be written as z_{hs}. The individual budget constraint is, therefore, $\sum_r p_r^* \sum_{s(r)} (z_{hs} - e_{hs})$, where summation over $s(r)$ indicates summation over all individual states s that occur in any social state leading to r, i.e. that are in the set $\Omega(r)$. Let $\#\Omega(r)$ be the number of social states in $\Omega(r)$. As $\Pi_{s|r} = r_s$ is the proportion of households in state s within the statistical state r, we can finally rewrite the budget constraint (4) of the household h as:

$$\#\Omega(r) \sum_r p_r^* \sum_s \#\Omega(r) \Pi_{s|r} (z_{hs} - e_{hs}) = 0. \tag{9}$$

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Using (2), the household's maximization problem can, therefore, be expressed as:

$$\max \sum_{s,r} \Pi_{sr} U^h\left(z_{hsr}\right) \text{ subject to (9)}$$

and the equilibrium allocation z_h^* by definition solves this problem. Similarly, we may rewrite the market clearing condition (5) as follows:

$$\sum_h \left(z_h^* - e_h\right) = \sum_h \left(z_{h\sigma}^* - e_{hs(\sigma)}\right) = 0, \quad \forall \sigma \in \Omega.$$

Rewriting the market clearing condition (5) in terms of statistical states r, and within each r, individual states s, we obtain:

$$\sum_s r_s H\left(z_{hr}^* - e_{sr}^h\right) = 0, \quad \forall r \in R \tag{10}$$

or equivalently:

$$\sum_s \Pi_{s|r} H\left(z_{hr}^* - e_{sr}^h\right) = 0, \quad \forall r \in R.$$

Using these relations, we now show that any Arrow–Debreu equilibrium allocation $z^* = \left(z_{hr}^*\right)$ is within the budget constraints (7) of the economy E_I for each $\sigma \in \Omega$, *provided that* for each $\sigma \in \Omega$ we add the income derived from a statistical security $A_r, r = r(\sigma)$, and, given $r(\sigma)$, the income derived from mutual insurance contracts $m_{sr}^h = m_{s(\sigma)r(\sigma)}^h, s = 1, \ldots, S$. We introduce A statistical securities and $I.A$ mutual insurance contracts in the general equilibrium economy with incomplete markets E_I. The quantity of the security A_r purchased by household h in statistical state r, when equilibrium prices are p^*, is:

$$a_r^{h*} = \sum_s \Pi_{s|r} p_r^*\left(z_{hr}^* - e_{hs}\right). \tag{11}$$

The quantity a_r^{h*} has a very intuitive interpretation. It is the expected amount by which the value of equilibrium consumption exceeds the value of endowments, conditional on being in statistical state r. So *on average*, the statistical securities purchased deliver enough to balance a household's budget in each statistical state. Differences between the average and each individual state are taken care of by the mutual insurance contracts. Note that (10) implies that the total amount of each security supplied is zero, i.e., $\sum_h a_r^{h*} = 0$ for all r, so that this corresponds to the initial endowments of the incomplete economy E_I. Furthermore, $\sum_r a_r^{h*} = 0$ by (9), so that each household h is within her/his budget in E_I.

We now introduce a mutual insurance contract as follows. The transfer made by individual h in statistical state r and individual state s, when prices are p_r^*, is:

$$m_{sr}^{h*} = p_r^*\left(z_{hr}^* - e_{hr}\right) - a_r^{h*}. \tag{12}$$

Note that, as remarked above, m_{sr}^{h*} is just the difference between the actual income-expenditure gap, given that individual state s is realized, and the expected income-expenditure gap a_r^{h*} in statistical state r, which is covered by statistical securities. In each statistical state r, the sum over all h and s of all transfers m_{sr}^{h*} equals zero, i.e. the insurance premia match exactly the payments: for any given r,

$$\sum_{h,s} H\Pi_{s|r} m_{sr}^{h*} = \sum_{h,s} H\Pi_{s|r} p_r^* \left(z_{hr}^* - e_{hs} \right) - \sum_h H a_r^{h*} \sum_s \Pi_{s|r}$$

$$= 0 \tag{13}$$

because $\sum_s \Pi_{s|r} = 1$. Therefore, the $\{m_{sr}^{h*}\}$ meet the definition of mutual insurance contracts. Finally, note that with N spot markets, A statistical securities $\{a_r\}$ and I mutual insurance contracts $\{m_{sr}^h\}$

$$p_r^* \left(z_{hr}^* - e_s^h \right) = m_{sr}^{i*} + a_r^{i*}, \quad \forall \sigma \in \Omega \text{ with } r(\sigma) = r, s = s(\sigma) \tag{14}$$

so that (7) is satisfied for each $\sigma \in \Omega$. This establishes that when all households have the same probabilities over social states, all Arrow–Debreu equilibrium allocation z^* of E can be achieved within the incomplete markets economy E_I when A securities and $I.A$ mutual insurance contracts are introduced into E_I, and completes the proof of the first part of the proposition dealing with common probabilities.

Consider now the case where the economy E is regular, different households in E have different probabilities over social states but have the same preferences, and $S = 2$. By Proposition 2, we know that within the set of equilibrium prices there is one p^* in which at all social states $\sigma \in \Omega(r)$ for a given r, the equilibrium prices are the same, i.e. $p_\sigma^* = p_r^*$. In particular, if E has a unique equilibrium (p^*, z^*), it must have this property. It follows from the above arguments that the equilibrium (p^*, z^*) must maximize (2) subject to (9). Now define the quantity of the security A_r purchased by a household in the statistical state r by

$$a_r^{h*} = \sum_s \Pi_{s|r}^h p_r^* \left(z_{hsr}^* - e_s^h \right) \tag{15}$$

and the mutual insurance transfer made by a household in statistical state r and individual state s, by

$$m_{sr}^{h*} = p_r^* \left(z_{hsr}^* - e_s^h \right) - a_r^{h*}. \tag{16}$$

As before, $\sum_r a_r^{h*} = 0$ and for any given r, $\sum_{h,s} \Pi_{s|r}^h H m_{sr}^{h*} = \sum_{h,s} r_s H m_{sr}^{h*} = 0$, so that the securities purchased correspond to the initial endowments of the economy E_I and at any statistical state the sum of the premia and the sum of the payments of the mutual insurance contracts match, completing the proof. $\square$

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Notes

1. A dealy viral disease.
2. Many were associated with hurricane Andrew which at $18 billion in losses was the most expensive catastrophe ever recorded. Some of the problems which beset Lloyds of London arose from underestimating environmental risks.
3. All consumption vectors are assumed to be non-negative.
4. See [16, p. 387, para. 1].
5. Defined formally below.
6. Related propositions were established by Malinvaud in an economy where all agents are identical, and risks are known.
7. The condition that all agents have the same preferences is not needed for this result. However, it simplifies that notation and the argument considerably. The general case is treated in the working papers from which this article derives.
8. The condition that all agents have the same preferences is not needed for this result, but simplifies the notation and the proof considerably. In the working papers from which this article derives, the general case was covered.
9. This follows from Dierker [11, p. 807] noting that his condition D is implied by our assumption that preferences are strictly increasing (see Dierker's remark following the statement of property D on p. 799).

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