

---

## Sustainable development: equal treatment of the present and the future?

---

Graciela Chichilnisky

Columbia Consortium for Risk Management,  
Columbia University,  
335 Riverside Drive, New York, NY 10025, USA  
E-mail: gc9@columbia.edu  
E-mail: chichilnisky1@gmail.com

**Abstract:** This article provides two axioms that capture the idea of sustainable development, and characterises the welfare criterion that they imply. It presents a formal theory of sustainable development, created by the author (Chichilnisky, 1996a, 1997). The axioms require that neither the present nor the future should play a dictatorial role in society's choices over time. Theorems 1 and 2 show that there exist sustainable preferences which satisfy these axioms and provide a full characterisation. Theorems 3 to 5 study a standard dynamical system representing the growth of a renewable-resource economy, give a 'turnpike' theorem, and exhibit the differences between sustainable optima and the ones according to discounted utilitarianism. (JEL 013)

**Keywords:** sustainable development; axioms; equal treatment for the present and future; intergenerational equity; green economics; long-run optimisation; dictatorship of the present and future; green golden rule; Weber-Fechner law; sustainable optimum.

**Reference** to this paper should be made as follows: Chichilnisky, G. (2010) 'Sustainable development: equal treatment of the present and the future?', *Int. J. Green Economics*, Vol. 4, No. 4, pp.346–359.

**Biographical notes:** Graciela Chichilnisky created the formal theory of sustainable development and the concept of basic needs (Chichilnisky, 1977a, 1977b, 1996a, 1996b, 1996c, 1997; Herrera et al., 1976). She is the Architect of the Kyoto Protocol's carbon market and acted as a Lead Author of the IPCC, which received the 2007 Nobel Prize. She is a Professor of Economics and Statistics and the Director of Columbia Consortium for Risk Management (CCRM) at Columbia University, New York.

---

### 1 A voracious use of resources

For the first time in history, human activity has reached levels at which it could alter the planet's climate and its biological mix. Industrialisation is the driving force. Energy used for production is obtained by burning fossil fuels, and leads to carbon emissions. The emissions generated since the Second World War could alter the earth's climate, although there is as yet no scientific agreement on the precise magnitude of the effects. Biologists see the loss of biodiversity during the last 50 years as one of the six largest incidents of destruction of life on the planet. Originating largely in the industrial countries, this

voracious use of resources has been accompanied by increasing discrepancies in resource consumption and welfare between industrial and developing countries. At the 1992 United Nations Earth Summit in Rio de Janeiro, sustainable development emerged as one of the most urgent subjects for international policy. One hundred and fifty participating nations endorsed UN Agenda 21, proposing as part of its policy agenda sustainable development based on the satisfaction of basic needs in developing countries. This development criterion was introduced by the author in 1976 in the ‘Bariloche model’ and given further impetus in 1987 when the Brundtland Commission (see Brundtland, 1987; Chichilnisky et al., 1998; Lauwers, 1993, 1997; Lowenstein and Elster, 1992; Lowenstein and Thaler, 1989) proposed that “sustainable development is development that satisfies the needs of the present without compromising the needs of the future”. But what is sustainable development?

Solow (1992) pointed out recently that discussion of sustainability has been mainly an occasion for the expression of emotions and attitudes, with very little formal analysis of sustainability or of sustainable paths for a modern industrial economy. Formal analysis requires valuation, and the economic value of a resource is usually derived from its contribution to utility. This suggests that the crux of the matter is how to describe value so that it does not underestimate the future’s interests and utilities, so that the future is given an equal treatment. This could be achieved in several ways. The challenge, however, is to develop economic theory which formalises this aim with the level of clarity and substance achieved by neoclassical growth theory, and with the practical scope of the current approach to cost benefit analysis that is based on neoclassical growth theory. This article provides two axioms that capture the idea of sustainable development, and characterises the welfare criterion that they imply. The axioms require that neither the present nor the future should play a dictatorial role in society’s choices over time.

### *1.1 Costs today, benefits tomorrow*

A well-known problem is that classical cost-benefit analysis (see Koopmans, 1968) discounts the future. It is therefore biased against policies designed to provide benefits in the very long run. A sharp example is the evaluation of projects for the safe disposal of waste from a nuclear power plant. Another is policies designed for the prevention of global warming. The benefits of both may be at least 50 to a hundred years into the future. The costs, however, are here today. In these cases, the inherent asymmetry between the treatment of present and future makes it hard to justify investment decisions that large numbers of individuals and organisations clearly feel are well merited. Recent experimental evidence sheds new light on the matter.

### *1.2 Experimental evidence*

Several experiments have measured how people value the long run [see, e.g., Lowenstein and Thaler (1989) and Cropper et al. (1994), the references in Lowenstein and Elster (1992) and Chanel and Chichilnisky (2009)]. Their findings clash with the traditional discounted approach. People are shown to value the present and the future differently, but not as the standard analysis would predict. The experimental evidence indicates that the present and the future are treated more even-handedly. Typically we do discount the future, but the trade-off between today and tomorrow blurs as we move into the future. Tomorrow acquires increasing relative importance as time progresses. It is as if we

viewed the future through a curved lens. The relative weight given to two subsequent periods in the future is inversely related to their distance from today. The period-to-period rate of discount is inversely related to the distance into the future. The experimental evidence shows that rate of discount between period  $t$  and period  $t + 1$  decreases with  $t$ . Interestingly, studies of human responses to sound summarised in the Weber-Fechner law, indicate similar responses to changes in sound intensity. The human ear responds to sound stimuli in an inverse relation to the initial stimulus.

How to explain this experimental evidence? How to make sense of our sensitivity to time and integrate it into a criterion of optimality? Several interesting alternatives to the discounted utility analysis have been proposed. So far none had reached the clarity and consistency of the discounted utilitarian criterion used in cost-benefit analysis, nor its analytical tractability. Prominent examples are the ‘overtaking criterion’ and Ramsey’s criterion. Both, however, are seriously incomplete, failing to rank many reasonable paths. The ordering induced by the overtaking criterion cannot be represented by a real valued function, making it impractical to use. As a result, they lack the corresponding ‘shadow’ prices to evaluate costs and benefits in an impartial fashion. These criteria therefore fail on practical grounds.

### *1.3 A criterion for sustainability*

This paper proposes simple axioms which capture the concept of sustainability, and derives the welfare criterion which they imply (see also Chichilnisky, 1996a, 1997). The criterion that emerges is complete, analytically tractable, and represented by a real valued function. In optimisation it leads to well-defined shadow prices which can be used for a ‘sustainable cost-benefit analysis’. The axioms provide internal consistency and ethical clarity. They imply a more symmetric treatment of generations in the sense that neither the ‘present’ nor the ‘future’ should be favoured over the other. They neither accept the romantic view which relishes the future without regards for the present, nor the consumerist view which ranks the present above all. The axioms lead to a complete characterisation of sustainable preferences, which are sensitive to the welfare of all generations, and offer an equal opportunity to the present and to the future. Trade-offs between present and future consumption are allowed. The existence and characterisation of sustainable preferences appears in Theorems 1 and 2. Theorem 1 shows that sustainable preferences are different from all other criteria used so far in the analysis of optimal growth and of markets. Theorem 3 studies a standard dynamical system representing the growth of a renewable resource. Sustainable preferences are shown to be a natural extension of the ‘equal treatment criterion’ for finitely many generations, in the sense that the optimal solutions for such preferences approach the ‘turnpike’ of an equal-weight finite horizon optimisation problem as the horizon increases. Theorems 3 and 4 also show that sustainable preferences match the experimental evidence in these cases, in the sense that they imply a rate of discount that decreases and approaches zero as time goes to infinity. Theorem 5 shows that sustainable optima can be quite different from discounted optima, no matter how small is the discount factor. Related axioms and results for the sustainable treatment of uncertainty and catastrophic risks are in Chichilnisky (2000, 2009a, 2009b, 2009c, 2010) and Chichilnisky and Eisenberger (2010).

## 2 Two axioms for sustainable development

The two following axioms are non-dictatorship properties. Axiom 1 requires that the present should not dictate the outcome in disregard for the future: it requires sensitivity to the welfare of generations in the distant future. Axiom 2 requires that the welfare criterion should not be dictated by the long-run future, and thus requires sensitivity to the present. To offer a formal perspective a few definitions are required.

For ease of comparison, I adopt a formulation which is as close as possible to that of the neoclassical model. Each generation is represented by an integer  $g$ ,  $g = 1, 2, \dots, \infty$ . An infinitely lived world obviates the need to make decisions contingent on an unknown terminal date. Generations could overlap or not. Indeed agents could be infinitely long-lived and evaluate development paths for their own futures.

Each generation  $g$  has a preference that can be represented by a utility function  $u_g$  for consumption of  $n$  goods, some of which could be environmental goods such as water, or soil, so that consumption vectors are in  $R^n$  and  $u_g: R^n \rightarrow R$ . The availability of goods in the economy is constrained in a number of ways, for example by a differential equation which represents the growth of the stock of a renewable resource, and/or the accumulation and depreciation of capital. Ignore for the moment population growth, although this issue can be incorporated at the cost of simplicity, but with little change in the results. The space of all feasible consumption paths is indicated by  $F$ .

We chose a utility representation so that each generation's utility function is bounded below and above. This choice is not restrictive: it was shown by Arrow (1964) that when ranking infinite streams of utilities as done here one should work with bounded utility representations since doing otherwise could lead to paradoxes. Utility across generations is assumed to be comparable. Each utility stream is a sequence of positive real numbers bounded by the number 1. The space of all utility streams is therefore contained in the space of all infinite bounded sequences of real numbers, denoted  $L$ . Our welfare criterion  $W$  ranks all feasible utility streams.

### 2.1 Sensitivity and completeness

The welfare criterion  $W$  is complete if it is represented by an increasing real valued function defined on all bounded utility streams. It is sensitive, or increasing, if whenever a utility stream  $a$  is obtained from another  $b$  by increasing the welfare of some generation, then  $W$  ranks  $a$  strictly higher than  $b$ .

### 2.2 The present

How to represent the present? Intuitively, the present is represented by all the utility streams which have no future: for any given utility stream  $a$ , its 'present' is represented by all finite utility streams which are obtained by cutting  $a$  off after any number of generations. Formally:

*Definition 1:* For any utility stream  $a$  and any integer  $K$  let  $a_k$  be the ' $K$ -cutoff' of the sequence  $a$ , the sequence whose coordinates up to and including the  $K$ th are equal to those of  $a$ , and zero after the  $K$ th.

*Definition 2:* The present consists of all feasible utility streams which have no future, i.e., it consists of the cutoffs of all utility streams.

### 2.3 The future

By analogy, for any given utility stream  $a$ , its 'future' is represented by all infinite utility streams which are obtained as the 'tail' resulting from cutting  $a$  off for any finite number of generations.

*Definition 3:* The  $K$ th tail of a sequence is the sequence whose coordinates up to and including the  $K$ th are zero and equal to those of the original sequence after the  $K$ th generation.

### 2.4 No dictatorial role for the present

*Definition 4:* A welfare function  $W$  gives a dictatorial role to the present, and is called a dictatorship of the present, if  $W$  is insensitive to the utility levels of all but a finite number of generations, i.e., if  $W$  is only sensitive to the 'cutoffs' of utility streams, and it disregards the utility levels of all generations from some generation on. For a mathematical definition see Chichilnisky (1996a, 1997).

The following axiom eliminates dictatorships of the present:

*Axiom 1:* No dictatorship of the present.

### 2.5 No dictatorial role for the future

*Definition 5:* A welfare function  $W$  gives a dictatorial role to the future, and is called a dictatorship of the future, if it is insensitive to the utility levels of any finite number of generations, or equivalently it is only sensitive to the utility levels of the 'tails' of utility streams.

A formal definition is provided in Chichilnisky (1996a, 1997). The welfare criterion  $W$  is therefore only sensitive to the utilities of 'tails' of streams, and in this sense the future always dictates the outcome independently of the present. The following axiom eliminates dictatorships of the future:

*Axiom 2:* No dictatorship of the future.

*Definition 6:* A sustainable preference is a complete sensitive preference satisfying Axioms 1 and 2.

## 3 Existence and characterisation of sustainable preferences

Why is it difficult to rank infinite utility streams? Ideally one would give equal weight to every generation. For example, with a finite number  $N$  of generations, each generation can be assigned weight  $1/N$ . But when trying to extend this criterion to infinitely many generations one encounters the problem that, in the limit, every generation is given zero weight.

What is done usually to solve this problem is to attach more weight to the utility of near generations, and less weight to future ones. An example is of course the sum of discounted utilities. Discounted utilities give a bounded welfare level to every utility stream which assigns each generation the same utility. Two numbers can always be

compared, so that the criterion so defined is clearly complete. However, the sum of discounted utilities is not even-handed: it disregards the long-run future. I show below that it is a dictatorship of the present.

Another solution is the criterion defined by the long-run average of a utility stream, a criterion used frequently in repeated games. However, this criterion is not even-handed either: it is biased in favour of the future and against the present. It is insensitive to the welfare of any finite number of generations.

Here, matters stood for some time. Asking for the two axioms together, the non-dictatorship of the present and the non-dictatorship of the future, as I do there, appears almost as if it would lead to an impossibility theorem. Not quite.

Let us reason again by analogy with the case of finite generations. To any finite number of generations one can assign weights which decline into the future, and then assign some extra weight to the last generation. This procedure, when extended naturally to infinitely many generations, is neither dictatorial for the present nor for the future. It is similar to adding to a sum of discounted utilities, the long-run average of the whole utility stream. Neither part of the sum is acceptable on its own, but together they are. This is Theorem 1 below. Theorem 2 proves that under regularity conditions this procedure gives a complete characterisation of all continuous sustainable preferences.

The first part of Theorem 1 establishes that the sum of a dictatorship of the present plus a dictatorship of the future is in fact neither. The first part is sensitive to the present, and the second is sensitive to the future. Furthermore such a sum admits trade-offs between the welfare of the present and of the future. It is represented diagrammatically in Figure 1 in Chichilnisky (1997), which shows the trade-offs between the present's and the future's utilities. The three axes represent the utility levels of generations 1, 2, and, figuratively,  $\infty$ . The two triangular planes represent two indifference surfaces. One gives more utility to generations 1 and 2, and under a dictatorship of the present these choices would prevail; however the second surface gives more utility to the long run, so that under certain conditions the second surface is chosen over the first. Theorem 1 makes this reasoning rigorous. It is proven in Chichilnisky (1997).

The second part of Theorem 1 shows that all known criteria of optimality used until now fail to satisfy the axioms postulated here. Therefore the sustainable preferences defined here perform a role satisfied by no previously used criterion.

What is perhaps more surprising is that the sustainable welfare criteria constructed here, namely the sum of a dictatorship of the present and one of the future, exhaust all the continuous utilities which satisfy my two axioms. This means that any continuous sustainable preference must be of the form just indicated. This is Theorem 2, proved in the Appendix of Chichilnisky (1997).

### 3.1 Existence of sustainable preferences

*Theorem 1:* There exists a sustainable preference  $W$  namely a preference which is sensitive and does not assign a dictatorial role either to the present or the future. It consists of a sum of a present discounted utility plus a preference assigning all the 'weight' to the long run future (an extension of the preference defined as  $\lim_{t \rightarrow \infty} (a_t)$ , see Chichilnisky (1996a, 1997). The following welfare criteria are not sustainable preferences:

- a the sum of discounted utilities, for any discount factor
- b Ramsey's criterion (see Ramsey, 1928)
- c the overtaking criterion (see von Weizacker, 1967)
- d  $\liminf$
- e long-run averages
- f Rawlsian rules (see Rawls, 1971)
- g basic needs.

*Proof:* In the Appendix of Chichilnisky (1997).

An intuitive explanation of this result follows. A preference defined by the sum of discounted utility and a limiting function at infinity is sustainable because it is complete, its first term is sensitive to the present, in fact it increases with increases in the welfare of every generation, and its second term is sensitive to the long-run future. The next step is to show why all other criteria fail the sustainability axioms proposed here.

- a The sum of discounted utilities is a dictatorship of the present because for every  $\epsilon > 0$ , there exists a generation  $N$  so that the sum of discounted utilities of all other generations beyond  $N$  is lower than  $\epsilon$  for all utility streams since all utilities are bounded by the number 1. Therefore there exists a generation  $N$  beyond which the utilities achieved by any generation beyond  $N$  do not count in the criterion  $W$ . This is true for any given discount factor (see Chichilnisky, 1996a, 1997).
- b The Appendix of Chichilnisky (1997) establishes that the Ramsey's criterion is incomplete; this derives from the fact that the distance to Ramsey's bliss path is ill-defined for many paths.
- c The Appendix of Chichilnisky (1997) establishes that the overtaking criterion is incomplete [see also Figure 2 of Chichilnisky (1997)].
- d and e  $\liminf$  and long-run averages are dictatorships of the future; furthermore the long-run average is also incomplete.
- f and g Rawlsian and basic needs criteria, are insensitive because they rank equally any two paths which have the same infimum even if one assigns much higher utility to many other generations.

### 3.2 A complete classification of sustainable preferences

It is possible to characterise all sustainable preferences. For this purposes additional conditions on the welfare criterion are introduced:  $W$  is continuous.<sup>1</sup> The following theorem gives a full characterisation to all sustainable criteria which are continuous.

A standard property of neoclassical analysis is that the rate of substitution between two generations which is generally dependent on their level of consumption is independent of their levels of utility. This is a widely used property: indeed the sum of discounted utilities, the most widely used welfare criterion, certainly satisfies it. A welfare criterion  $W$  satisfying this property is called independent. In consistency with

neoclassical analysis, we therefore assume independent and continuous welfare criteria. A theorem in Chichilnisky (1996a, 1997) decomposes a sustainable criterion into the sum of two functions. The first is a discounted utility with a variable discount rate and the second is a generalisation of long-run averages or a lim inf, and is called a 'purely finitely additive' measure. For definitions and examples see Appendix of Chichilnisky (1997).

#### **4 Are sustainable preferences reasonable?**

We saw that sustainable preferences emerge from well-defined and uncontroversial axioms. But how do they fit economic intuition and empirical evidence? In Chichilnisky (1996a, 1997), we showed that sustainable preferences fit well our economic intuition about finite horizon optimisation, and that they also fit the empirical evidence rather well.

##### *4.1 A turnpike theorem*

Our economic intuition is grounded on finite horizons. Life on earth will certainly be of finite duration, although it is difficult to determine its final date. It is therefore important to determine whether sustainable preferences are merely an artefact of infinite horizons, or are reasonable within a finite world.

Sustainable preferences can be seen as a suitable generalisation to infinite horizons of an intuitively appealing criterion for finite horizons, one which values all generations equally, thus providing 'equal treatment' (see Chichilnisky, 1997). Indeed, for a general class of dynamic optimisation problems, the limit of the optimal solution according to a sustainable preference has two interesting properties:

- 1 it is the 'green golden rule' (see Beltratti et al., 1995), that is, the configuration of the economy giving the maximum sustainable utility level
- 2 as the finite horizon increases, the optimal solutions of equal treatment finite horizon problems spend an increasing amount of time progressively closer to the limit of the path which is optimal according to sustainable preferences.

In other words, the optimum according to sustainable preferences determines a direction in which finite horizon equal treatment optima increasingly move as the horizon increases. We refer to this property as a 'turnpike' property.

Consider a typical problem of optimising a sustainable preference over a constraint imposed by the dynamics of a renewable stock. A renewable stock  $s$ , grows over time  $t$  according to its own biological dynamics, and some of it is extracted for consumption in each period. The utility depends on consumption and the level of the stock, as in for example Krautkraemer (1985) or Heal (1995).<sup>2</sup> To study the asymptotic properties of a maximum for this problem it is useful to introduce the following definition (see also Beltratti et al., 1995):

*Definition 7:* The green golden rule  $g^*$  is a stationary path which achieves the maximum utility level which is sustainable forever.

*Theorem 3:* The optimal path for renewable resource problem with a sustainable preference exists if and only if the discount rate approaches zero in the limit, in which

case the optimum is the ‘turnpike’ of finite horizon problem in which each generation is treated equally.

By contrast, the discounted utilitarian optimum does not have the turnpike property for equal treatment optima, for any positive (fixed) discount rate the optimal path for a positive discount rate is uniformly bounded away from  $g^*$  and so from the turnpike of the equal treatment optima. For a zero discount rate, the discounted utility problem has no solution.

*Proof:* See Heal (1995).

Figure 3 in Chichilnisky (1997) illustrates the results.

#### *4.2 Empirical evidence matches sustainable preferences*

Sustainable preferences help explain the empirical evidence on time preferences. Recent empirical evidence on time preferences clashes with standard discounted utility maximisation. There is a growing body of empirical evidence that suggests that the discount rate which people apply to future projects depends upon, and declines with, the futurity of the project. See for example, Lowenstein and Thaler (1989) or Cropper et al. (1994). Over relatively short periods up to perhaps five years, they use discount rates which are higher even than commercial rates – in the region of 15% or more. For projects extending about ten years, the implied discount rates are closer to standard rates – perhaps 10%. As the horizon extends the implied discount rates drop, to in the region of 5% for 30 to 50 years and down to of the order of 2% for 100 years. The evidence for these statements is still tentative, and more research is needed to document fully how people trade off the future against the present. However, our framework for inter-temporal optimisation has an implication for discounting that rationalises a behaviour that hitherto has been found irrational. The evidence is consistent with the solutions obtained from optimising sustainable preferences in the context of a renewable resource, in the sense that an optimum exists if and only if the discount rate falls over time, as shown above and in Chichilnisky (1997).

#### *4.3 Sustainability and the Weber-Fechner law*

Well-known results from natural sciences establish that human responses to a change in a stimulus are non-linear, and are inversely proportional to the existing level of the stimulus. For example, the human response to a change in the intensity of a sound is inversely proportional to the initial sound level: the louder the sound initially, the less we respond to a given increase. This is an example of the Weber-Fechner law, which can be formalised in the statement that human response to a change in a stimulus is inversely proportional to the pre-existing stimulus. This law has been found to apply to human responses to the intensity of both light and sound signals. The empirical results on discounting cited above suggest that something similar is happening in human responses to changes in the futurity of an event: a given change in futurity (e.g., postponement by one year) leads to a smaller response in terms of the decrease in weighting, the further the event already is in the future. In this case, the Weber-Fechner law can be applied to responses to distance in time, as well as to sound and light intensity, with the result that the discount rate is inversely proportional to distance into the future.

In our economic problem such a discount factor meets all of the conditions required for the existence of sustainable optima, as shown in Chichilnisky (1997). A discount factor has now an interesting interpretation: for example, the replacement of  $t$  by  $\log t$  implies that we are measuring time differently: by equal proportional increments rather than by equal absolute increments. This is consistent with the approach taken in, for example, acoustics, where in response to the Weber-Fechner law sound intensity is measured in decibels which respond to the logarithm of the energy content of the sound waves. In general, non-constant discount rates can be interpreted as a non-linear transformation of the time axis.

## **5 Differences between standard and sustainable optima**

### *5.1 Renewable and exhaustible resources*

Chichilnisky (1997) explored the difference between sustainable and discounted utilitarian optima in the case of dynamic problems with renewable and exhaustible resources.

Sustainable preferences typically lead to more conservation, to a larger long-run stock, than the discounted utilitarian framework with the same discount rate. In particular, the final stock will be higher, the initial consumption level lower, and the initial shadow price higher (see also Heal, 1995).

## **6 Sustainable optima that are far from discounted optima**

We have shown that sustainable preferences are substantially different from other welfare criteria which have been used in the literature. They are also different in practice, for example, the optimal solutions of problems which maximise sustainable preferences are substantially different from the optimal solutions to discounted problems. To answer this question, Chichilnisky (1997) compares problems which face the same constraints, but each of which maximises different welfare criteria. The purpose is to explore what difference this makes in practice. A problem which maximises a sustainable preference will be called a sustainable problem. If the welfare criterion is a discounted sum of utilities we call this a discounted problem. The corresponding solutions are called sustainable optima and discounted optima.

Can one always approximate a sustainable optimum by paths which optimise discounted problems? Or even better: can one always approximate a sustainable optimum by a sequence of paths which approximates the solutions of a discounted problem? In Chichilnisky (1997) we provide a negative answer to these two questions. It is not always possible to approximate sustainable optima by paths which approach discounted optima. Sustainable optima and discounted optima can be far apart.

*Example 1:* Consider an economy which uses trees as a necessary input to production or consumption; without this input the economy's utility is zero. The dynamics of tree reproduction requires that unless in the first  $N$  periods the economy preserves a minimum number of trees, the species becomes extinct after  $K + N$  periods, in which case there is zero utility at every period from there on. The economy's feasible set of utility streams is

described then as follows: a minimum investment denoted  $\epsilon > 0$  is required during each of the first  $N$  periods to ensure that the utility levels in all periods from the  $(N + K)$ th on, is above zero. Once this threshold is reached, then all utility levels in each period after the  $(N + K)$ th exceed the value  $E$ . Then for every discount factor, there is an  $N, K$  for which the sum of discounted utilities is maximised at a path which leads to the eventual elimination of the forest. Instead, for a sustainable preference which gives sufficient weight to the long run the optimum will keep the forest alive and yielding a minimum utility level  $E$  forever. Therefore, the two optima are apart by at least  $E$ ; any sequence of paths which approaches the discounted optimum will not approach the sustainable solution.

## 7 Sustainability and value

The notion of value derived from sustainable preferences is rather distinctive. Paths which are optimal under sustainable preferences may not maximise present discounted value according to any standard price system. Therefore, environmental resources with a large value in the long run, may not appear valuable under a standard notion of present value profit maximisation. Indeed, Chichilnisky (1997) exhibits a sustainable optimum which does not maximise discounted present value for any standard price system.

## 8 Conclusions

This article presents a formal theory of sustainable development. It is based on axioms that capture the idea of sustainability, and characterises the sustainable preferences that they imply. Neither the present nor the future should be dictatorial. The article analysed other criteria used in the literature, and found that they do not satisfy our axioms. Discounted utility fails to satisfy the non-dictatorship of the present. This agrees with the viewpoint of many practitioners, who have pointed out the inadequacy of discounted utility for analysing sustainable growth? Basic needs and the overtaking criteria are incomplete as they fail to compare many reasonable alternatives. This decreases their value as tools for decision making. Ramsey's criterion has a similar drawback: it is defined as the integral of the distance to a 'bliss' utility level, but this integral is often ill-defined.

The sustainable preferences proposed here and characterised above resolve these problems. From the practical point of view, they satisfy two desirable criteria: they fit our intuition of finite horizon problems, because in important examples they have a turnpike property with respect to equal treatment finite horizon problems. In addition, they fit rather well empirical observations that indicate that people's perceptions of the future imply lower discount rates as time progresses. Important classes of dynamic problems have a solution according to sustainable preferences only if the implied discount rates are decreasing through time.

The article and its predecessors (Chichilnisky, 1996a, 1997) showed that sustainable preferences give rise to optimal solutions which are different from those obtained by discounted optimisation criteria. A path which is optimal under a sustainable preference may not be approximated by paths which approximate discounted optima.

The notion of value derived from sustainable preferences is distinctive. Paths which are optimal under sustainable preferences may not maximise value according to any standard price system. Therefore, environmental resources with a large value in the long run, may not appear valuable under a standard notion of profit maximisation.

These results may help to disentangle the apparent contradictions in values which were discussed in the introduction. Governments and international organisations appear seriously concerned about global environmental problems which lie so far into the future that with current discounted utility measures they do not lead to substantial economic loss. The axioms for sustainable preferences proposed here may help resolve this contradiction. Discounted profit maximisation and sustainability lead to different value systems. Some trade-offs are possible, but the two values are not the same. The empirical evidence we have today is more in favour of sustainable preferences than discounted utility. Solow (1992) has proposed that sustainability should allow intergenerational trade-offs, but no generation should be favoured over any other. This standard is met by sustainable preferences when applied to the 'present' and to 'future' generations. The long run does matter and so does the short run. Sustainable preferences define shadow prices for sustainable optima, which can be used for project evaluation and for the characterisation of optimal solutions.

It remains to understand the concern for the long-run future which is observed in practice, and which appears formalised in the axioms proposed here and their implied preferences. Nobody alive today, not even their heirs, has a stake on the welfare of 50 generations into the future. Yet, many humans care about the long-run future of the planet, and the results of this paper indicate that axioms which formalise this concern are not altogether unacceptable. One may then ask: whose welfare do sustainable preferences represent?

Perhaps, an answer for this riddle may be found in a wider understanding of humankind as an organism who seeks its overall welfare over time. Such proposals have been advanced in the concepts of a 'selfish gene', or, more practically, in Eastern religions which view the unity of humankind as a natural phenomenon. If such unity existed, humankind would make up an unusual organism, one whose parts are widely distributed in space and time and who is lacking a nervous system on which the consciousness of its existence can be based. Perhaps the recent advances in information technology, with their global communications and processing reach, are a glimmer of the emergence of a nervous system from which a global consciousness for humankind could emerge.

### **Acknowledgements**

This work was carried out with the support of the US Air Force Office of Scientific Research in Arlington Virginia, USA.

### **References**

- Arrow, K.J. (1964) 'Aspects of the theory of risk bearing', Yrjii Jahansson Lectures, Yrjo Jahonssonin Sdatio, Helsinki.
- Beltratti, A., Chichilnisky, G. and Heal, G.M. (1995) 'The green golden rule: valuing the long run', *Economics Letters*, Vol. 49, No. 2, pp.175-179.

- Brundtland, G.H. (1987) *The U.N. World Commission on Environment and Development: Our Common Future*, Oxford University Press, Oxford.
- Chanel, O. and Chichilnisky, G. (2009) 'The influence of fear in decisions: experimental evidence', *Journal of Risk and Uncertainty*, Vol. 39, No. 3, pp.271–298.
- Chichilnisky, G. (1977a) 'Economic development and efficiency criteria in the satisfaction of basic needs', *Applied Mathematical Modeling*, Vol. 1, No. 6, pp.290–297.
- Chichilnisky, G. (1977b) 'Development patterns and the international order', *Journal of International Affairs*, Vol. 31, No. 2, pp.275–304.
- Chichilnisky, G. (1982) 'Social aggregation and rules and continuity', *Quarterly Journal of Economics*, Vol. 97, No. 2, pp.337–352.
- Chichilnisky, G. (1994) 'Social diversity, arbitrage and gains from trade: a unified perspective on resource allocation', *American Economic Review*, Vol. 84, No. 2, pp.427–434.
- Chichilnisky, G. (1996a) 'An axiomatic approach to sustainable development', *Social Choice and Welfare*, Vol. 13, No. 2, pp.231–257.
- Chichilnisky, G. (1996b) 'Updating Von Neumann Morgenstern axioms for choice under uncertainty', *Invited Presentation, Proceedings of a Workshop on Catastrophic Environmental Risks*, 9–11 June 1996, The Fields Institute for Mathematical Sciences, University of Toronto, Canada.
- Chichilnisky, G. (1996c) 'Development and global finance: the case for an international bank for environmental settlements', *United Nations Development Program Publication No. 10*, UNESCO, New York, available at <http://www.chichilnisky.com>; books and writings; also (1997) in Dragun, A.K. and Jakobsson, K.M. (Eds.): *Sustainability and Global Environmental Policy: New Perspectives*, Chapter 13, pp.249–278, Edward Elgar, Cheltenham.
- Chichilnisky, G. (1997) 'What is sustainable development', *Land Economics*, Vol. 73, No. 4, pp.467–491.
- Chichilnisky, G. (2000) 'An axiomatic approach to choice under uncertainty with catastrophic risks', *Resource and Energy Economics*, Vol. 22, No. 3, pp.221–231.
- Chichilnisky, G. (2009a) 'The topology of fear', *Journal of Mathematical Economics*, Vol. 45, Nos. 11–12, pp.807–816.
- Chichilnisky, G. (2009b) 'The limits of econometrics: NP estimation in Hilbert spaces', *Econometric Theory*, Vol. 25, No. 4, pp.1070–1086.
- Chichilnisky, G. (2009c) 'Avoiding extinction: equal treatment of the present and the future', *Economics: The Open-Access, Open Assessment E-Journal*, Vol. 3, No. 2009-32.
- Chichilnisky, G. (2010) 'The foundations of statistics with black swans', *Journal of Probability and Statistics, Special Issue on Actuarial and Financial Risks: Models, Statistical Inference, and Case Studies (AFR)*.
- Chichilnisky, G. and Eisenberger, P. (2010) 'Asteroids: assessing catastrophic risks', *Journal of Probability and Statistics*.
- Chichilnisky, G. and Heal, G. (1983) 'Necessary and sufficient conditions for a resolution of the social choice paradox', *Journal of Economic Theory*, Vol. 31, No. 1, pp.68–87.
- Chichilnisky, G., Heal, G. and Vercelli, S. (1998) *Sustainability, Dynamics and Uncertainty*, Kluwer Academic Publishers, Dordrecht, Boston, London.
- Cropper, M.L., Aydede, S.K. and Portney, P. (1994) 'Preferences for life saving programs: how the public discounts time and age', *Journal of Risk and Uncertainty*, Vol. 8, pp.243–265.
- Heal, G. (1995) *Valuing the Future: Economic Theory and Sustainability*, Book Manuscript; also Working Paper University of Oslo, 1995, Later published as Heal, G. (2002) *Valuing the Future*, Columbia University Press, New York.
- Herrera, A., Scolnik, H., Chichilnisky, G. et al. (1976) *Catastrophe or New Society: A Latin American World Model (The Bariloche Model)*, International Development Research Center, Ottawa, Canada.

- Koopmans, T. (1968) *Three Essays on the State of Economic Science*, McGraw-Hill Inc., New York.
- Krautkraemer, J. (1985) 'Optimal growth, resource amenities and the preservation of natural environments', *Review of Economic Studies*, Vol. 52, pp.153–170.
- Lauwers, L. (1993) 'Infinite Chichilnisky rules', Discussion paper, Katholik Universitaet, Leuven, March, *Economics Letters*, April 1993, Vol. 42, No. 4, pp.349–352.
- Lauwers, L. (1997) 'A note on weak x-Chichilnisky rules', *Social Choice and Welfare*, Vol. 14, No. 2, pp.357–359.
- Lowenstein, G. and Elster, J. (Eds.) (1992) *Choice Over Time*, Russell Sage Foundation, New York.
- Lowenstein, G. and Thaler, R. (1989) 'Intertemporal choice', *Journal of Economic Perspectives*, Vol. 3, No. 4, pp.181–193.
- Ramsey, F. (1928) 'A mathematical theory of saving', *Economic Journal*, Vol. 38, pp.543–559.
- Rawls, J. (1971) *A Theory of Justice*, Bellknap Press, Cambridge, MA.
- Solow, R. (1992) 'An almost practical step toward sustainability', *Lecture on the Occasion of the Fortieth Anniversary of Resources for the Future*, October, Washington, DC.
- von Weizacker, C.C. (1967) 'Lemmas for a theory of approximate optimal growth', *Review of Economic Studies*, January, Vol. 34, pp.143–151.

## Notes

- 1 Continuity has played a useful role in social choice theory in the last ten years, in effect replacing the axiom of independence of irrelevant alternatives and allowing a complete characterisation of domains in which social choice exists (Chichilnisky 1982, 1994; Chichilnisky and Heal, 1983). A similar role is found here for continuity.
- 2 We are assuming the stock of the resource to be an argument of the utility function, so that the resource is in the category of environmental assets such as forests, landscapes, biodiversity, etc., which provide services and value to human society via their stocks as well as via a flow of consumption.