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The Influence of Fear in Decisions: Experimental Evidence

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Abstract

This article studies decisions made under conditions of fear, when a catastrophic outcome is introduced in a lottery. It reports on experimental results and seeks to compare the predictions of the expected utility (EU) framework with those of a new axiomatic treatment of choice under uncertainty that takes explicit account of emotions such as fear (Chichilnisky, 1996, 2000, 2002). Results provide evidence that fear influences the cognitive process of decision-making by leading some subjects to focus excessively on catastrophic events. Such heterogeneity in subjects' behavior, while not consistent with EU-based functions, is fully consistent with the new type of utility function implied by the new axioms.

Keywords: decision under risk, losses, catastrophic event, fear, probability weighting function.

JEL Classification: C91, D81

Emotions undeniably influence our behavior both as consumers and as human beings, inducing changes in the chemistry of our bodies and our brains. Psychologists and neurobiologists have developed tools to study their consequences on human behavior, and market researchers have even exploited them for commercial purposes. Yet few attempts have been made to incorporate the role of emotions in the field of economic theory.[1]

Typical emotions influencing behavior include fear as well as anger, joy, disgust, admiration, guilt, pride, shame, envy. Yet economists do not seem to have devoted as much attention to the emotion of fear as they have to guilt, envy, regret or shame. Indeed, Elster's (1998) survey on "Emotions and Economic Theory" hardly mentions fear. However, recent empirical research on the Value of a Prevented Fatality (VPF) provides evidence of a "dread effect" that differs depending on the type of death at stake, in the spirit of Sunstein's (1997) proposal to add a "bad death" premium for types of death that are particularly frightening or dreadful. Chilton et al. (2006) for instance estimate the contribution of dread effects to elicited VPF's to be small for pedestrian accident cause (19%) and automobile accident cause (18%) but to be predominant for rail accident cause (98%) and fire in a public place cause (97%). Viscusi (2009) found that the prevention of terrorism deaths is valued by respondents almost twice as highly as preventing natural disaster deaths; he argues that terrorism risks are perceived vividly due to the 9/11 attacks and involve a substantial element of dread.

Fear is indeed an emotion that plays a crucial role when individuals face choices under uncertainty involving catastrophic events, i.e. rare events where outcomes are associated with a huge disutility. When facing such catastrophic events - like earthquake, terror-

[1] A first step was taken with models accounting for the fact that individuals project themselves after the time of the decision (regret theory, see Bell, 1982; Loomes and Sugden, 1982; disappointment theory, see Loomes and Sugden, 1986; Gul, 1991); then with models introducing the possibility that emotions felt during the decision-making process also affect the decision (see Loewenstein et al., 2001); and more recently with models in which a Proportion of Emotion (poe) felt during the cognitive decision-making process is explicitly included in the weighting function that determines the behavior (see Reid, 2006).

ist attacks, major nuclear accident, the consequences of global warming - individuals are likely to experience fear and adopt behaviors that appear as irrational. Sunstein (2003) provides evidence that subjects show unusually strong reactions to low-probability catastrophes especially when their emotions are intensely engaged. This "probability neglect" is also explored in Sunstein and Zeckhauser (2008) for fearsome risks, as well as the damaging results of these overreactions on individual behavior and government regulations. Unfortunately, the dominant theory that formalizes decision-making under uncertainty - the standard expected utility (EU) theory of Von Neumann and Morgenstern - is based on rational behavior alone, ignoring emotions, and is unable to handle such behaviors.

This article seeks to compare the predictions of the EU framework with those of a new axiomatic treatment of choice under uncertainty (see Chichilnisky 2000, 2002, 2009; and Appendix 1) that takes explicit account of changes in answers arising when fear is an issue. The distinguishing feature of this new theory is that it negates one important axiom in the Von Neumann and Morgenstern theory: the Monotone Continuity (MC) axiom. This paper tests experimentally whether this one axiom is logically negated or is not. When this axiom is satisfied, individuals always behave according to the EU framework. When it is satisfied sometimes and sometimes not (faced with catastrophic outcomes), the MC axiom is logically negated and individuals change their responses in a predictable fashion. This is what we actually obtain in our experiments. The experimental results are indeed consistent with the optimization of a new type of utility function implied by the new axioms - such as the sum of expected utility and extremal utility values - since in some cases the responses are consistent with expected utility while in others they are influenced by extremal events and driven by fear (Chichilnisky 2000, 2002, 2009).

Being able to measure on the same scale a catastrophic event and a variety of non-catastrophic events is crucial to explore apparently irrational behaviors. We chose a hypothetical scenario involving a kidnapping, in which the outcome is "being locked up in a room with no chance of escaping, being freed or communicating (with relatives, friends...), with nothing interesting to do". Although this situation is bearable and not catastrophic for short periods, it becomes a catastrophic event when it lasts for a very long period. Our setting controls for misunderstanding or very strong overweighting of small probabilities that could drive subjects' behavior when facing rare events. Moreover, we avoid parametric estimations that might lead to debatable results. 50 subjects participated in the experiment. They were randomly assigned to one of four groups of 10 to 14 subjects (either graduate or Ph.D. students) in a classroom. Seated alone at a table, each subject individually answered a written questionnaire in four parts, with permission to ask any question aloud. The trade-off (TO) method (initiated by Wakker and Deneffe, 1996) was used to elicit each subject's utility and probability weighting functions before and after the introduction of a catastrophic event.

The main results are the following. All behavior is in accordance with the EU framework when there are no catastrophic events. However, 52% of the subjects do not answer according to the EU framework when the catastrophic event is introduced, leading to an apparent lack of rationality. All these behaviors are compatible with the new axioms for choice under uncertainty reported here (Appendix 1) that negate the MC axiom. Methodologically, experimental results cannot provide a definitive test for any given theory, no matter how thorough the experiments. However the experimental results presented here suffice to reject the predictions of the EU framework in situations where the subjects are subject to fear, and they are consistent with the axioms of Chichilnisky (2000, 2002, 2009) taking fear into consideration.

The remainder of the paper proceeds as follows. Section 1 presents the three-step elicitation method and Section 2 describes how it was implemented. The results of the experiment are given in Section 3, and discussed in Section 4. Section 5 concludes. Appendix 1 summarizes the axioms of the new theory based on fear, as well as some of its main results.

1 Experimental design

We design a three-step experiment that encompasses EU as a restriction of more general classes of models allowing for subjective transformation of consequences and probability: Rank Dependent Expected Utility (RDEU) models. The founding EU theory formalized by Von Neumann and Morgenstern (1944) was already barely standing up to empirical tests as early as 1953 and Allais' articles, and generalizations of EU were being proposed by the mid-1970's. We therefore consider that subjects may rely on the RDEU model when answering, which assumes a weakening of the independence axiom. Steps 1 and 2 allow us to classify subjects according to the model they mentally referred to when making choices under uncertainty (lotteries in the domain of losses with non-catastrophic outcomes). The emotion of fear is then introduced in Step 3 by making one outcome catastrophic. We analyze individual answers to check whether the rationalities observed when subjects face lotteries involving catastrophic and non-catastrophic events are similar or not.

1.1 Background

Uncertainty is described by a system that is in one of several **states**, indexed by the real numbers with a standard Lebesgue measure. In each state a monotonically increasing continuous **utility function** $u : R^n \to R$ ranks the **outcomes**, which are described by vectors in R^n. When the probability associated with each state is given, a description of the utility achieved in each state is called a **lottery:** a function $f : R \to R$. **Choice under uncertainty** means the ranking of lotteries. **Axioms** for choice under uncertainty describe natural and self-evident properties of choice, like ordering, independence and continuity. On the basis of these axioms of choice a crucial result of the standard theory created by Von Neumann and Morgenstern (Arrow, 1971) established that individuals optimize the ranking of lotteries $W_{EU}(f)$ according to an *expected utility function.* The **expected utility** of a lottery f is a ranking defined by $W_{EU}(f) = \int_{x \epsilon R} f(x) d\mu(x)$ where μ is a measure with an integrable density function $\phi(.)$ that belongs to the space of all measurable and integrable functions on R so $\mu(A) = \int_A \phi(x)dx$, where dx is the standard Lebesgue measure on R. The ranking $W_{EU}(.)$ is a continuous linear function that is defined by a countably additive measure μ.

Attempts have been made to deal with theoretical difficulties raised by the EU model, by weakening or relaxing some axioms. Quiggin (1982) initiated the Rank Dependent Expected Utility (RDEU) theory in which individuals rank outcome according to the utility function u **and** a transformation $w(.)$ of the outcomes' cumulative probabilities.[2] Since RDEU theory is generally consistent with the observed behavioral patterns that violate the independence axiom (with an appropriate choice of a probability transformation function, $w(.)$), we choose to express the EU model as a special restriction of the RDEU model. The ranking of a lottery f is defined by $W_{RDEU}(f) = \int_{x \epsilon R} f(x)w'(1-\mu(x))d\mu(x)$ where $w(.)$ is a continuous strictly increasing function from $[0, 1]$ to $[0, 1]$, with $w(0) = 0$ and $w(1) = 1$. In the following, the wording RDEU hence covers EU as a special case where the $w(.)$ function is equal to the identity function.

In contrast, Chichilnisky (2000, 2002, 2009) recently defined a different set of axioms that take explicit account of fear. In practice, the "Monotone Continuity Axiom" of the standard theory is replaced by its logical negation, an axiom requiring sensitivity to rare events (Axiom 2, Appendix 1). An individual who acts according to these new axioms optimizes a different type of utility function, which is not an EU function. All the rankings of lotteries that satisfy the new axioms were identified in Chichilnisky (2000) (see also Appendix 1). It emerges that the utilities implied by the new axioms are quite different from EU: they are the sum of an EU function and a second function solely focused on extremal events - and which represents the fear they induce. The experimental results

[2] The Cumulative Prospect Theory (Tversky and Kahneman, 1992) is more general than RDEU since it allows probability weighting functions to differ in gains ($w^+(.)$) and in losses ($w^-(.)$). However, the CPT and the RDEU coincide in our experiment as lotteries are in the domain of losses only, and we do not present CPT.

appear to be consistent with such a sum of utilities, since the behavior of the subjects sometimes agrees with expected utility and sometimes does not agree - which is predictable when maximizing such a sum and since the new axioms negate the "Monotone Continuity Axiom" as shown in Chichilnisky (2009).

1.2 First step: eliciting the utility function

We restrict to lotteries with at most two outcomes that only differ in one dimension at a time (i.e., either probability or outcome), in order to keep the cognitive task as simple as possible. We adapt the trade-off (TO) method (see for instance Wakker and Deneffe, 1996; Fennema and Van Assen, 1999; Abdellaoui, 2000) to elicit the Von Neumann-Morgenstern utility function of a subject. This method is robust and not affected by non-linear probability weighting. We elicit a sequence of outcomes $x_0, \ldots, x_n$, called **a standard outcome sequence**, using a sequence of choices between lotteries.

The first outcome in the sequence, x_1, is the value that makes the subject indifferent between lottery $(x_0, p; r, 1-p)$ and lottery $(x_1, p; R, 1-p)$, where:

- x_0 (the first point of the sequence) as well as r and R, are fixed outcomes.
- $(x_1, p; R, 1-p)$ is a lottery with probability p of winning x_1 and probability $1-p$ of winning R,
- $0 > u(R) > u(r) > u(x_0)$, where $u(.)$ is a utility function that ranks the outcomes (utility is in fact disutility since we only consider losses),
- p is fixed during the first step.

Under RDEU, the indifference between $(x_0, p; r, 1-p)$ and $(x_1, p; R, 1-p)$ implies:

$$w(p)u(x_0) + [1 - w(p)]u(r) = w(p)u(x_1) + [1 - w(p)]u(R)$$

where $w(.)$ is the probability transformation function ($w(.)$ is the identity function under EU).

Then, the second outcome in the sequence, x_2, is the value that makes the subject indifferent between lottery $(x_1, p; r, 1-p)$ and lottery $(x_2, p; R, 1-p)$. The process goes on until the last outcome of the sequence, x_n, is obtained, when the subject is indifferent between $(x_{(n-1)}, p; r, 1-p)$ and lottery $(x_n, p; R, 1-p)$.

According to RDEU, the elements of the standard outcome sequence elicited by each subject are equally spaced in terms of subjective utility. Hence, for each indifference within the outcome sequence, we have:

$$u(x_i) - u(x_{(i+1)}) = \{[1 - w(p)]/w(p)\}[u(R) - u(r)] \qquad i = 0, ..., n-1$$

Provided r, R and p do not change during the elicitation process, the following equalities hold:

$$u(x_0) - u(x_1) = u(x_1) - u(x_2) = \ldots = u(x_i) - u(x_{(i+1)}) = \ldots = u(x_{(n-1)}) - u(x_n)$$

As the utility function is unique up to a positive affine transformation, we adopt the usual normalization and set $u(x_0) \equiv 0$ and $u(x_n) \equiv -1$ and therefore:

$$u(x_i) = -i/n \qquad i = 0, \ldots, n \tag{1}$$

This elicitation method has been proved to be very robust (in particular to the propagation of errors in subjects' answers, see Bleichrodt and Pinto, 2000).

1.3 Second step: eliciting the probability weighting function

This step provides the subjective weights each subject gives to objective probabilities. It is sometimes used in the literature to discriminate between EU, RDEU and CPT by studying - for both gains and losses - the way subjects weight utility and probability. The main objective of this paper is not to characterize probability weighting functions but to elicit exact probabilities associated with catastrophic and non-catastrophic outcomes. Consequently, the Probability Equivalent (PE) method was chosen: each subject determines the value p_i

such that s/he is indifferent between lotteries $(x_n, p_i; x_0, 1-p_i)$ and $(x_i, 1)$, for different values of x_i $(i = 1, ..., n-1)$ in the standard outcome sequence.[3]

Under RDEU, these indifferences allow us to compute the subjective transformation of each p_i as:

$$\begin{aligned} & w(p_i)u(x_n) + [1 - w(p_i)]u(x_0) = u(x_i) \qquad i = 1, ..., n-1 \qquad (2) \\ \Rightarrow \quad & w(p_i) = [u(x_i) - u(x_0)]/[u(x_n) - u(x_0)] \end{aligned}$$

As the x_i are chosen in the standard outcome sequence, they are equally spaced in terms of subjective utility and the expression $u(x_i) - u(x_{(i-1)})$ is constant $\forall\ i = 1, ..., n$. Hence (1) and (2) lead to:

$$w(p_i) = i/n \qquad i = 1, ..., n-1.$$

Under EU, this reduces to $p_i = i/n$ as $w(.)$ is the identity function. Note that, because subjects are asked to assess p_i in the survey (and not $w(p_i)$), inverse images $w^{-1}(.)$ are required to establish the probability weighting function.[4] Subjects will be classified according to their weighting function by comparing the probability p_i (i.e. $w^{-1}(i/n)$) and the corresponding value $w(p_i)$ (i.e. $w[w^{-1}(i/n)] \equiv i/n$).

The main drawback of the PE method is that the response scale changes (from outcomes to probabilities). This deserves attention when designing the experiment, as this scale change might distort the preference elicitation (see Delquié, 1993; or Tversky, Sattah and Slovic, 1988).

[3] The two alternative methods - certainty equivalent and trade-off - are not relevant here since both require the probabilities to be chosen by the experimenter. In the former, each subject determines the certainty equivalent value CE_{p_j} that makes him/her indifferent between lotteries $(x_k, p_j; x_i, 1-p_j)$ and CE_{p_j}, where x_k and x_i are elements of the standard outcome sequence and p_j are different fixed values. In the latter, each subject determines the outcome Z_{p_j} such that s/he is indifferent between lotteries $(x_i, p_j; x_j, 1-p_j)$, and $(x_h, p_j; Z_{p_j}, 1-p_j)$ for different values of p_j, with $x_h > x_i > x_j$ elements of the standard outcome sequence.

[4] CE and TO methods would provide images of $w(1-p_j)$ for given $(1-p_j)$ through parametric estimations.

1.4 Third step: introducing fear

The key idea is to repeat the second step by replacing x_n with a catastrophic event denoted by x_{CAT}. Each subject is now asked to determine the value p_{CAT} such that s/he is indifferent between the lotteries $(x_{CAT}, p_{CAT}; x_0, 1 - p_{CAT})$ and $(x_i, 1)$, for a specified i in the outcome sequence $(i = 1, ..., n - 1)$. Thus, we have:

$$w(p_{CAT})u(x_{CAT}) + [1 - w(p_{CAT})]u(x_0) = u(x_i) \qquad i = 1, ..., n - 1$$

$$\Rightarrow \quad w(p_{CAT}) = [u(x_i) - u(x_0)]/[u(x_{CAT}) - u(x_0)]$$

The p_{CAT} value elicited by each subject is compared to the expected value given his/her previously elicited $u(.)$ and $w(.)$. If they strongly differ, this means that his/her behavior is not compatible with the EU framework. Because it ignores the fear generated during the cognitive process, this framework cannot properly account for the disutility associated with the catastrophic event. The change in subjects' behavior is due to the logical negation of the Monotone Continuous axiom (as proved in Chichilnisky, 2009) and is compatible with her axiomatic treatment.

2 The experiment

2.1 General settings

The questionnaire was tested in a pilot session involving 6 subjects, to fine-tune wording and procedure. Then, 50 subjects participated in the final experiment, being randomly assigned to one of four groups of 10 to 14 subjects. All subjects, either graduate or Ph.D. students in Social Sciences, Economics, Mathematics or Science and Engineering, were paid a lump sum of 15 euros (about $20). The experiment, lasting between 60 and 90 minutes depending on the group, was not computerized. Assembled in a classroom, the subjects were seated one per table and individually answered a written questionnaire in four parts, with permission to ask any question aloud (see Appendix 2 for the script and typical elicitation questions). This procedure meant that experimenters had to transfer the subject's numeric values for the standard outcomes obtained in Step 1 to the questionnaires relative to Steps 2 and 3 before handing the latter out.

The choice of a suitable outcome is crucial, since it should enable discrimination between a catastrophic and a non-catastrophic event with the same unit of measurement. We chose a hypothetical scenario involving a kidnapping, in which the outcome is "being locked up in a room with no chance of escaping, being freed or communicating (with relatives, friends...), with nothing interesting to do". Indeed, although this situation is bearable and not catastrophic for short periods, it becomes a catastrophic event when it is going to last for a very long period. It is precisely this ability to measure on the same scale a catastrophic event and a variety of non-catastrophic events (i.e. different periods) that appears useful in exploring the influence of fear in stated behaviors.

The detention periods are expressed in months (with a minus sign since they generate utility losses). Based on the pilot session, we set $R = -1$, $r = -2$, $x_0 = -3$, $x_{CAT} = -480$ (i.e. 40 years) and $p = 0.33$. Although the elicitation procedure does not depend on p, this value has been found in many studies to be the least affected by subjective distortion of probability (e.g. Tversky and Fox, 1995; Prelec, 1998) and is used in many empirical studies (Wakker and Deneffe, 1996; Fennema and Van Assen, 1999; Etchart-Vincent, 2009).

2.2 Step 1

Each session starts by reminding subjects that they are not allowed to communicate or interact with each other. The experimenters hand out (and read aloud) a first questionnaire that explains the whole procedure and the trade-off method. Subjects are told that they have to choose between two lotteries: ZUW and ZOW. These words avoid potential framing effects when names involve a mental association, like A and B for instance. Two practice questions involving two choices between two lotteries are given, and subjects by themselves then answer the questions eliciting the utility function.

The standard outcome sequence for utility involves $n = 6$ choices and determines $x_1, ..., x_6$. Each subject fills in this part of the questionnaire by writing the value x_1 that makes him/her indifferent between lottery ZUW (-3, 0.33; -2 , 0.67) and lottery ZOW ($x_1, 0.33; -1, 0.67$). S/he then copies down this x_1 value to create the next choice between a new lottery ZUW (x_1, 0.33; $-2, 0.67$) and a new lottery ZOW ($x_2, 0.33; -1, 0.67$) and so on (see Appendix 2).

Once all subjects have registered their six values $x_{1,}..., x_6$, the experimenters collect the first questionnaire and hand out a new questionnaire eliciting general socioeconomic information and identifying subjects' attitudes, beliefs and knowledge concerning risk. While subjects fill in this part, the experimenters prepare for the elicitation of the probability weighting function by copying the standard outcome sequence $x_1, ..., x_6$ elicited from each subject in Step 1 onto the third (Step 2) and fourth (Step 3) parts of the questionnaire.

2.3 Steps 2 and 3

The probability weighting function is elicited through a set of choice questions. Each subject is asked to determine probabilities p_i, $i = 1, ..., 5$, that make him/her indifferent between lottery $(x_6, p_i; x_0, 1 - p_i)$ and lottery $(x_i, 1)$, where the x_i belong to the standard outcome sequence. To limit the effect of the change in the response scale and help subjects understand probabilities, we follow the recommendations of previous studies on the way to represent unfamiliar risk with small likelihood of occurring (which is the case with our outcome: "being locked up in a room"). Kunreuther, Novemsky and Kahneman (2001), for instance, "highlight the need to give individuals enough context to draw on their own experiences and well-developed risk perceptions, if we are to ask them to evaluate an unfamiliar risk which has a small likelihood of occurring". Calman and Royston (1997) think that "better ways are required for presenting risk magnitudes in a digestible form, and a logarithmic scale provides a basis for a common language for describing a wide range of risks. Various 'dialects' of this language - visual, analogue, and verbal scales - could help with grasping different risk magnitudes."

We thus use several representations to help subjects correctly interpret a (very) small probability:[5]

- a (logarithmic) graphic scale is provided in the questionnaire, adapted from Calman and Royston (1997) and Corso, Hammitt and Graham (2001). It shows on the same sheet risk magnitude (with a stepsize of 10) from 1:1 to 1:10,000,000,000; the corresponding expression in percentage with "community analogies" also represented by a pictogram and examples of various events involving probabilities of the same magnitude (see Appendix 3);

- a physical representation with "probability analogies" represented by sheets of paper laid out in the room from 1:1 (one sheet to one) to 1:100,000 (one sheet to 40 boxes of 2,500), with a stepsize of 10;

- a visual representation with different surfaces displayed on a blackboard from 1:1 (the whole blackboard surface, i.e. 6,200 square inches) to 1:100,000 (a dot of 0.062 square inch), with a stepsize of 10.

Whether or not using several visual representations is constructive and helps subjects process the risk information reliably is discussed in the following section.

Subjects fill in the third part of the questionnaire, which the experimenters collect before handing out the last part, in an envelope to be opened by the subjects. The catastrophic event (Step 3) is introduced by setting x_6 to $x_{CAT} = -480$ in the lottery ZUW and asking each subject the value p_{CAT} such that s/he is indifferent between lotteries $(x_{CAT}, p_{CAT}; x_0, 1 - p_{CAT})$ and $(x_3, 1)$.

3 Results

Subjects were recruited from various disciplines but required to "have basic skills in probability". As the procedure was self-administered and not computer-assisted, real time de-

[5] This also limits the likelihood of discontinuities of the probability function due to difficulties in valuing and dealing with extreme (i.e. close to 0 or 1) probabilities.

tection of subjects' inconsistencies was not possible. We therefore begin by testing whether the experiment was properly understood and followed by subjects.

3.1 Evidence of subjects' correct processing of experimental information

Checking how reliably subjects process the experimental information is crucial before considering inconsistencies in their answers to the EU framework. For this purpose, we successively look for inconsistencies in elicitations of the standard outcome and sequence of probabilities, then analyze questions eliciting the easiness of answering questions in the different parts of the experiment and the usefulness of the aids provided to represent probability.

3.1.1 Testing for subjects' inconsistencies

Because our concern is with individual behavior under conditions of fear and the remainder of our analysis is actually performed at subject level, we carry out straightforward tests of inconsistencies at subject level rather than at aggregate level.

Testing the outcome understanding

We use the standard outcome sequence $x_0, \ldots, x_6$ elicited by each subject in Step 1. The difference between two successive step sizes of the standard utility sequence, $\delta_i = x_i - x_{(i-1)}$ for $i = 1...6$, should be strictly negative. If not, the monotonically increasing property of the utility function is violated since an adverse outcome $x_{(i-1)}$ in the lottery ZUW $(x_{(i-1)}$, $0.33; -2, 0.67)$ would be preferred to a less adverse outcome x_i in the lottery ZOW $(x_i, 0.33; -1, 0.67)$. We found 6 subjects to have at least one $\delta_i \geq 0$ and we excluded them from the remainder of the analysis.[6]

Testing understanding of probability

We use the probability sequence $p_1, \ldots, p_5$ elicited in Step 2. Indeed, each subject

[6] Examples of inconsistencies in the $(x_0, ...x_6)$ standard outcome sequence are for instance $(-3, -1.5, -1.5, -1.5, -2, -1, -2)$ or $(-3, -4.5, -6, -5, -4.5, -3, -3)$.

gives the value p_i that makes him/her indifferent between $(x_6, p_i; x_0, 1 - p_i)$ and the sure outcome $(x_i, 1)$. As the sure outcome becomes decreasingly attractive as i increases, this implies that the lottery $(x_6, p_i; x_0, 1-p_i)$ is less attractive than the lottery $(x_6, p_{(i-1)}; x_0, 1-p_{(i-1)})$. Hence a probability p_i of adverse outcome x_6 makes a lottery less attractive than a comparable lottery with lower probability $p_{(i-1)}$ of adverse outcome if $\eta_i = p_i - p_{(i-1)} > 0$ for $i = 2...5$. Among the 44 remaining subjects, 2 have at least one $\eta_i \leqslant 0$ and are consequently excluded.[7] The final sample is now 42.

3.1.2 Qualitative self-assessments of the experimental procedure

Let us analyze two sets of qualitative self-assessment questions at the end of the questionnaire, regarding the perceived difficulty of answering the questions in the experiment and the usefulness of the aids provided to represent probability.

The first set of questions asks subjects to give a mark that best represents the level of difficulty they experienced answering the questions in the different parts of the experiment. We reproduce in Table 1 the mean value and the standard deviation of subjects' marks for each of the items proposed.

[INSERT TABLE 1 ABOUT HERE]

On average, the 42 subjects found the experiment not difficult since most of the means are below 5. It is encouraging to note that the lowest marks are obtained for choosing a probability involving a 40-year detention period (4.17) and for the change of the response scale from outcome to probability (4.43). This result confirms that subjects encountered no additional difficulty with these two tasks, crucial to the experiment.

The third column of Table 1 provides the same statistics for the eight subjects previously excluded due to major inconsistencies and shows that all the means are larger than 5

[7] The two inconsistencies w.r.t. the p_i sequence are (0.2, 0.05, 0.25, 0.4, 0.8) and (0.333, 0.25, 0.7, 0.9, 0.95).

(between 6.38 and 7.88). Interestingly enough, the means of the excluded subjects are all significantly higher than the corresponding mean of the sample, according to formal two-tailed t-test for equality of means (see last column of Table 1 for the corresponding p-values). The fact that these subjects felt the experiment difficult certainly explains their inability to answer properly.

The second set of questions asks subjects to give a mark that best represents the usefulness of the aids provided to represent probabilities. We reproduce in Table 2 the mean value, the standard deviation and the range of subjects' marks for each of the aids proposed.

[INSERT TABLE 2 ABOUT HERE]

The most useful way to present risk information appears on average to be the probability scale expressed in percentage (4.24), while the physical and visual representations (2.31 and 2.69) are the least useful. This is not surprising since percentages are common in everyday life whereas physical representations are unusual! However, looking at which types of representation were awarded the highest marks, we find that each of them appears very useful (mark 9 or 10) to at least some subjects, a finding that validates our decision to use different representations.

Lastly, we compute the difference between the highest and the lowest mark by subject to explore whether using several visual representations is constructive. The average difference is 2.98 and the range 0-9, which confirms that using various visual representations was indeed useful to subjects.

Overall, these results indicate that the 42 subjects process the experimental information provided on the lottery, probability and risk representation in a reliable manner.

3.2 Subjects' behavior in a non-catastrophic setting

3.2.1 Utility functions

Aggregate Data

Figure 1 presents the elicited aggregate functions (on mean and median data) along with some examples of subjects' transformations. The mean data exhibits slight convexity whereas median data is very close to linearity. Hence, it seems that although the outcome is non-standard ("being locked up in a room"), results are comparable to what has been observed elsewhere with losses of money or life expectancy, for instance.

[INSERT FIGURE 1 ABOUT HERE]

Individual data

We proceed as Abdellaoui (2000), Fennema and Van Assen (1999) or Etchart-Vincent (2004), classifying individuals according to the difference between two successive step sizes of the standard utility sequence. We then use the previously computed δ_i, $i = 1...6$, to calculate $\Delta_i = \delta_i - \delta_{(i-1)}$ for $i = 2...6$. For each subject, we obtain 5 values for Δ_i. Because of response error, we consider the following classification criterion: concavity (resp. convexity, linearity) holds when at least 3 out of 5 values are positive (resp. negative, null). When no conclusion is possible, we class the function as "mixed" shape.

There are 6 subjects with at least 3 negative values for Δ_i, 32 subjects with at least 3 zero values for Δ_i, 1 subject with 3 positive values for Δ_i and 3 subjects with mixed values for Δ_i.[8] Clearly, the dominant pattern is linearity: 35 out of 42 subjects (83.3%) exhibit a linear utility function. This result is not a surprise and is frequently found in experiments involving the domain of losses. Indeed, to actually observe a diminishing sensitivity to losses (convexity) may require a longer detention period.

3.2.2 Probability weighting function

Attention must be paid to the analysis and the interpretation of the probability weighting function as the subjects' assessed values (p_i) are inverse images $w^{-1}(i/n)$ for $0 < i < n$

[8] After close examination, subjects with mixed values appear to roughly follow a linear path with minor deviations towards both concavity and convexity. They are thus classified as having a linear utility function in the following.

(Abdellaoui, 2000).

Aggregate Data

Figure 2 presents four individual probability weighting functions as an illustration - two contrasted transformations of all probabilities (an overweighting and an underweighting), and two inverse S-shaped transformations - along with the transformation obtained with mean and median data.

[INSERT FIGURE 2 ABOUT HERE]

For the median data, there are only very limited departures from an identity weighting function with a slight overweighting for $p < 0.5$. For the mean data, the greatest - but still small - overweighting occurs between 0.5 and 1. For the mean data, this means that the transformation of probabilities has more impact near certainty than near impossibility (as found for instance in Abdellaoui, 2000; or Tversky and Fox, 1995). However, these authors found a significant underweighting in the 0.5-1 region (consistent with the CPT for losses, which claims risk aversion for low probability and risk seeking for high probability), whereas we found very limited evidence of risk aversion in the same region.

Individual data

Our main interest is neither in aggregate data nor in the whole probability range but in individual behavior on small probabilities. Indeed, Step 3 introduces a catastrophic outcome that subjects are expected to associate with a small probability. We classify the probability weighting function of subjects on the range 0-0.5 and obtain 19 concave, 5 identity, 17 convex and 1 mixed function. It appears that the number of subjects overweighting small probabilities is roughly equivalent to the number of subjects underweighting small probabilities in the 0-0.5 range (which explains the limited departures observed on aggregate data), and that 5 subjects do not transform probabilities at all.

3.2.3 Classification of subjects

Subjects are classified according to the shape of their utility and probability weighting functions. They are considered as behaving according to:

- "strict EU" if the probability weighting function is the identity function $w(p) = p$,
- "RDEU" if the utility function is convex or linear[9] and the probability weighting function is not the identity function on the range 0-1,
- "a non-identical weighting probability function" if the utility function is concave and the probability weighting function is not the identity on the range 0-1.

Table 3 shows that the dominant pattern (85.71%) is RDEU. 5 subjects (11.91%) behave strictly as predicted by EU and one (2.38%) according to the predictions of a non-identical weighting probability function. Overall, 97.62% of the subjects behave according to the EU framework.

[INSERT TABLE 3 ABOUT HERE]

3.3 Subjects' behavior when fear is introduced

When they opened the envelope and discovered the lottery that involves a 40-year detention period, many of the subjects reacted with a muffled exclamation of surprise. However, none decided to give up the experiment by not answering the question.

3.3.1 Assessing subjects' compatibility with the EU framework

Each of the subjects provides a p_{CAT} value that makes him/her indifferent between $(x_{CAT}, p_{CAT}; x_0, 1 - p_{CAT})$ and $(x_3, 1)$, where x_3 is subject-specific but always below

[9] Linearity is also considered since the magnitude of the outcome might be too small to observe convexity in the domain of losses.

$x_0 \equiv -3$.[10] Under RDEU, indifference implies that p_{CAT}^{RDEU} satisfies the following:

$$
\begin{aligned}
& w(p_{CAT}^{RDEU})u(x_{CAT}) + [1 - w(p_{CAT}^{RDEU})]u(x_0) = u(x_3) \\
\Rightarrow \quad & w(p_{CAT}^{RDEU}) = [u(x_3) - u(x_0)]/[u(x_{CAT}) - u(x_0)] \qquad (3) \\
\Rightarrow \quad & p_{CAT}^{RDEU} = w\left\{[u(x_3) - u(x_0)]/[u(x_{CAT}) - u(x_0)]\right\}^{-1} \qquad (4)
\end{aligned}
$$

If the subjects' stated value p_{CAT} is compatible with his/her former behavior in the non-catastrophic setting, the spread between p_{CAT} and p_{CAT}^{RDEU} should be small. In order to assess this spread with no parametric estimations, we proceed as follows.

First, although x_0 (common to all subjects) and x_3 belong to the subjects' standard outcome sequence, x_{CAT} (i.e. -480) does not. The assessment of p_{CAT}^{RDEU} requires a parametric estimation of $u(-480)$ that we avoid by assuming $u(.)$ to be linear which gives $w[(3 + x_3)/(3 - 480)]^{-1}$. Indeed, the utility function significantly deviates from linearity for 7 out of 42 subjects only: 1 towards concavity and 6 towards convexity. As the linear utility assumption leads to an underestimation (overestimation) of p_{CAT}^{RDEU} for the convex (concave) utility function, special attention will be devoted to these seven subjects when measuring the spread between p_{CAT} and p_{CAT}^{RDEU}.

Second, the subjective treatment of probabilities affects the spread. Compared to identity, very marked concavity of $w(.)$ for small p_{CAT} probabilities, i.e. strong risk aversion, reduces p_{CAT}^{RDEU}. On the other hand, convexity of $w(.)$ for small p_{CAT} probabilities increases p_{CAT}^{RDEU}. We assume identity of $w(.)$ as a benchmark, and carefully analyze the answers of the 19 subjects exhibiting a concave $w(.)$ probability weighting function for small probabilities, to determine whether or not the concavity drives the result, i.e. explains a small observed p_{CAT}.

Third, we account for the subjects' errors and/or approximations when eliciting p_{CAT} by allowing for uncertainty in the spread assessment. We thus choose to consider that a subject:

[10] Remember that subjects exhibiting major inconsistencies have been discarded.

- behaves according to the EU framework if the ratio $p_{CAT}/p_{CAT}^{RDEU} \in [0.2, 5]$, i.e. if his/her answer is between five times lower and five times higher than expected,[11]

- underweights p_{CAT} in a way not compatible with the EU framework if this ratio is lower than 0.2,

- overweights p_{CAT} in a way not compatible with the EU framework if this ratio is higher than 5.

3.3.2 Results

The classification of subjects' answers according to their utility and probability weighting functions are provided in Table 4 as well as in Table 5 in aggregate.

[INSERT TABLE 4 ABOUT HERE]

Ten subjects answer $p_{CAT} = 0$ (including the only subject with a concave utility function and a subject with a convex one), which implies indifference between $(-3, 1)$ and $(x_3, 1)$, although x_3 is between -4.5 and -18 (months) depending on the subject! Thus, while they behave rationally during Steps 1 and 2 and perfectly understand probabilities, they give an answer that is not compatible with any concept of rationality under risk, since it implies equivalence between $u(x_3)$ and $u(-3)$, i.e. with the normalization adopted earlier that $-0.5 \sim 0$! The introduction of fear apparently leads to irrational behavior.

Nine subjects give a p_{CAT} that underestimates the expected value in a way not compatible with their previous behavior (even accounting for the underestimation of low probabilities for the 6 subjects with concave probability function on the range 0-0.5).[12] Their

[11] Setting the bounds at [0.1,10] changes the classification for 2 subjects only.

[12] In addition to these 6 subjects, 13 other subjects exhibited a concave probability function for low probabilities in Step 2 that could lead to a very small p_{CAT}^{RDEU}. Among them, 5 answered $p_{CAT} = 0$, 6 answered a p_{CAT} value that is compatible with the EU framework and 2 a value that implies overweighting. Consequently, the influence of the concavity of the probability function does not bias our results towards more underweighting.

respective p_{CAT} lie in the range (0.00001, 0.001), whereas expected values under the EU-based framework were in the range (0.007, 0.021). The subjective treatment of the disutility associated with the catastrophic outcome implies a p_{CAT} lower than the value that could be expected to prevail in the RDEU models.

[INSERT TABLE 5 ABOUT HERE]

Seventeen subjects (including one with a convex utility function) answer in a way compatible with their behavior in the non-catastrophic setting (Step 2): EU or RDEU. Of these, the five who answer exactly according to EU do not change their reasoning (most of them give the fraction that makes the two lotteries exactly equivalent!). For these subjects, the fear associated with the catastrophic event is not sufficient to significantly affect their behavior.

The six remaining subjects (including three with a convex utility function) overweight p_{CAT} in a way not compatible with RDEU models, with respective p_{CAT} as follows: 0.05, 0.1, 0.2, 0.4, 0.5 and 0.5. The first three values deal with subjects with convex probability weighting function for p_{CAT} in the range 0-0.166. They are hence risk-seeking and their answers could in some sense be considered as representing their subjective transformation of a low probability, and hence as being compatible with RDEU. The last three answers are highly questionable since they deviate greatly from the rational pattern underlying EU and RDEU rankings.

4 Discussion

This study sheds light on the way fear considered as an emotion may influence choices under uncertainty. We compare answers by subjects presented with lotteries involving non-catastrophic and catastrophic outcomes. The results presented in the previous section require further comment, especially on how they can be interpreted as rational.

Let us first consider the 19 subjects who underestimate the p_{CAT}^{RDEU} associated with the catastrophic event to an extent not compatible with their previous behavior. Although RDEU is unable to predict their answers, it could be argued that a probability weighting function differing according to the emotion might fit. Rottenstreich and Hsee (2001) for instance introduced - in the CPT setting - the effect of fear (and hope) in the probability weighting function. Hence, the degree of over- or underweighting of a probability, since it depends on the strength of the emotion associated with the outcome evaluated, may explain changes between Step 2 and Step 3, due to the introduction of fear. In the same vein, the Proportion of Emotion approach (Reid, 2006) also explicitly models changes in the emotional weighting a subject gives to outcomes in a given context by introducing a measurement of the subject's emotional response (physiological or self-assessed) in the probability weighting function.

These models could admittedly explain the behavior of 9 out of the 19 subjects who underestimate p_{CAT}^{RDEU}, but in no way that of the 10 remaining subjects who answer $p_{CAT} = 0$. The fact that they fail to associate a probability, however tiny, with the catastrophic outcome in a lottery involving a 3-month detention that should make them indifferent to a much longer duration with certainty, violates the requirements of the EU-based framework. The only theory that explains their behavior is the theory proposed in Chichilnisky (2000, 2002) because it allows subjects to sometimes satisfy the MC axiom of the Von Neumann and Morgenstern theory and sometimes not. This logical negation of the MC axiom is equivalent to the sensitivity to rare events. As a consequence, subjects change their answers in a predictable fashion when the MC axiom is not satisfied. We are not aware of any other theory of choice under uncertainty which negates the MC and is still continuous.

This theory accounts for the specificity of extremal events by using the ranking of lotteries (see Appendix 1, Eq. (5)). Under this ranking, the answers of the 10 subjects answering $p_{CAT} = 0$ imply that $u(x_3) \sim \lambda u(-3) + (1 - \lambda) \langle f, \varphi_2 \rangle$ where $\langle f, \varphi_2 \rangle$ stands for the fear associated with the catastrophic event. This equivalence holds for every couple that verifies $(1-\lambda)^{-1} = -2\langle f, \varphi_2 \rangle$ as long as $\lambda < 1$, and the illusion of "irrational behavior" according to RDEU rankings no longer exists. The answers of the 9 other subjects who underestimate p_{CAT}^{RDEU} could also be analyzed via the weight devoted to the $(1-\lambda)\langle f, \varphi_2 \rangle$ component.

Let us now consider the 17 subjects whose answers were compatible with the RDEU approach. When the value of λ is set close to 1, the Topology of Fear (TF) is able to explain their behavior: the catastrophic event did not induce fear sufficient to deserve a greater weight in the ranking criteria.

Let us now finally consider the three subjects whose p_{CAT} was respectively 0.4, 0.5 and 0.5. These answers can be interpreted as reflecting a confused state of mind, leading them to choose a roughly 50/50 probability. For instance, one subject wrote in open comments "I was completely terrified by the last lottery *(i.e., Step 3)*, and I first chose 100%, then 0%, then I was not at all sure what I should do". This seems to support the reasons given in Chichilnisky (2000) for the domination of the amygdala in the decision-making process: ignorance and over-complex processing. Under conditions of fear, the cortex gives up and the whole primitive brain (including the amygdala) takes over and gives an "extremal" response. Indeed, when a subject **must** choose a p_{CAT}, the fear inspired by the 40-year detention outcome leads him/her to consider the whole probability range (0,1) and then choose a value at random: roughly the middle of the range.

To sum up, the ranking based on the TF is able to explain all the behavior of all the subjects - including that which are apparently irrational - thanks to the weight $(1-\lambda)$ devoted to the catastrophic and frightening outcome.

5 Conclusion

This paper presents an experiment on choices under uncertainty when a catastrophic outcome is introduced, and interprets the results taking account of the effect of the fear generated during the cognitive decision-making process. It provides evidence of heterogeneity in subjects' behavior since some answers are not compatible with EU-based standard theories explaining choices under risk.

On the one hand, 22 subjects answer according to the EU-based framework in non-catastrophic situations but not when a catastrophic outcome is introduced. They either give a null probability weight leading to an apparent lack of rationality (10 subjects), a too-small weight (9 subjects) or appear to be totally confused by giving a value at random (3 subjects).

On the other hand, 20 subjects answer according to EU-based standard theories in both catastrophic and non-catastrophic situations and their choices are not over-affected by the introduction of fear.

All behaviors are however perfectly compatible with the ranking implied by the TF: when there are no catastrophic events, the ranking is only based on the EU term, whereas when there is an event perceived by the subject as frightening, extra weight is given to this catastrophic event.

This experiment constitutes a very preliminary attempt to explore how cognitive processes could be affected during choices under uncertainty. Further laboratory experiments are necessary and should consider whether certain respondents' characteristics explain sensitivity to fear or should attempt to induce more markedly non-EU based behavior (by using more markedly catastrophic outcomes).

Appendix 1 - A new axiomatic treatment of choice under uncertainty: the topology of fear (TF)

We briefly present below the new axiomatic treatment of choice under uncertainty and the corresponding rankings introduced in Chichilnisky (2000, 2002, 2009).

Let us define an **event** E as a set of states, and E^c as the set of states of the world not in E (i.e. the complement of the set E). A **monotone decreasing** sequence of events $\{E^i\}_{i=1}^{\infty}$, is a sequence for which for all i, $E^{i+1} \subset E^i$. If there is no state in the world common to all members of the sequence, $\bigcap_{i=1}^{\infty} E^i = \varnothing$ and $\{E^i\}$ is called a **vanishing sequence**.

The key difference between the standard axiomatic treatment and that proposed by Chichilnisky is the way continuity is defined, i.e. the notion of "closeness" that is used. In Arrow (1971), two lotteries are **close** to each other when they have different consequences in **small** events. Arrow defines the Monotone Continuity as follows: "An event that is far out on a *vanishing sequence* is '**small**' by any reasonable standards". Hence, two lotteries that differ in sets of small enough Lebesgue measure are very close to each other.

Chichilnisky (2000, 2002) proposed a different definition of closeness. She used a L_∞ sup norm that is based on extremal events: two lotteries f and g are close when they are uniformly close almost everywhere (a.e.), i.e. when $\sup_R \mid f(t) - g(t) \mid < \epsilon$ a.e. for a suitable small $\epsilon > 0$. As a consequence, some catastrophic events are small under Arrow's definition - which averages events - but not necessarily under Chichilnisky's - which accounts for extremal events. Since her topology focuses on situations involving extremal events, such as (natural) catastrophes, it makes sense to call this the **topology of fear (TF)** (see Chichilnisky, 2009).

The core here is that her definition of closeness is more sensitive to rare events than Arrow's. This higher sensitivity constitutes the second of her three axioms restated below, which must be satisfied by a ranking W to evaluate lotteries:

Axiom 1: The ranking $W : L_\infty \to R$ is linear and continuous on lotteries.

Axiom 2: The ranking $W : L_\infty \to R$ is sensitive to rare events.

Axiom 3: The ranking $W : L_\infty \to R$ is sensitive to frequent events.

When one considers a family of subsets of events containing no rare events, for example when the Lebesgue measure of all events contemplated is bounded below, EU (and more generally all approaches relying on the EU axiomatic treatment, like RDEU for instance) satisfies all three Axioms. However, Chichilnisky (2009) has proved that sensitivity to catastrophic events requires the non satisfaction of the Monotone Continuity Axiom. Because the EU-based framework always satisfies the MC Axiom, it fails to explain the behavior of individuals facing catastrophic events The fact that the MC axiom is sometimes satisfied and sometimes not is precisely the "logical negation" of the MC Axiom. Formally:

Theorem 1: The Monotone Continuity Axiom implies the logical negation of sensitivity to rare events, namely Axiom 2. (see proof in Chichilnisky, 2009)

Do some decision criteria satisfy all three Axioms in the presence of rare events? Yes, if we modify expected utility, adding another component called 'purely finitely additive' elements of L^*_∞[13] that embodies the notion of sensitivity for rare events. The only acceptable rankings W under the three axioms above are a convex combination of L_1 function *plus a* purely finitely additive measure putting all weight on extreme or rare events as stated in the Theorem below:

[13] The space L^*_∞ is called the 'dual space' of L_∞, and is known to contain two different types of rankings $W(.)$, (i) integrable functions in $L_1(R)$ that can be represented by countably additive measures on R, and (ii) 'purely finitely additive measures' which are not representable by functions in L_1 (Chichilnisky, 2000), and are not continuous with respect to the Lebesgue measure of R.

Theorem 2: A ranking of lotteries $W : L_\infty \to R$ satisfies all three axioms 1, 2 and 3, if and only if there exist two continuous linear functions on L_∞, ϕ_1 and ϕ_2 and a real number λ, $0 < \lambda < 1$, such that:

$$W_{TF}(f) = \lambda \int_{x \epsilon R} f(x)\phi_1(x)dx + (1 - \lambda)\langle f, \phi_2 \rangle \tag{5}$$

where $\int_R \phi_1(x)dx = 1$, while ϕ_2 is a purely finitely additive measure. (see proof in Chichilnisky 1996, 2000, 2002).

The first term in (5) is similar to EU. The density $\phi_1(x)$ defines a countably additive measure that is absolutely continuous with respect to the Lebesgue measure.[14] The second term is of a different type: the operator $\langle f, \phi_2 \rangle$ represents the action of a measure $\phi_2 \epsilon L^*_\infty$ that differs from the Lebesgue measure in placing full weight on rare events. Remember that ϕ_2 cannot be represented by an L_1 function.

Hence, when the Monotone Continuity Axiom is logically negated, individuals change their responses in a predictable fashion, using the decision criterion proposed in Theorem 2.

Appendix 2 - Script and typical elicitation questions for the three steps

INSTRUCTIONS

The aim of the experiment is to study the way you make choices. During each of the sessions, you will have to answer questions dealing with fictitious situations. However, we ask you to answer as if the situations were real. Please take your time answering.

Some of the questions require a major cognitive effort and we suggest you carefully consider the question before answering. There are no right or wrong answers, since it is your own

[14] A measure is called absolutely continuous with respect to the Lebesgue measure when it assigns zero measure to any set of Lebesgue measure zero; otherwise the measure is called singular.

opinion that drives your behavior. The sum of money that will be given to you at the end of the experiment is a flat rate of 15 euros and does not depend on your answers.

In each session, you will have to choose between two lotteries: ZUW and ZOW. These lotteries involve both probabilities and outcomes. For each question, you should fill in the only empty space by choosing a value such that it makes the two options equivalent in your judgement. The figure you choose must make you indifferent between the two lotteries.

In Step 1, you are going to choose the length of time that makes you indifferent between two lotteries. In Step 2 and 3, you are going to choose the probability that makes you indifferent between the two lotteries.

Each of these lotteries is represented as a pie. Each piece of the pie corresponds to an outcome, and the size of the piece stands for the probability of occurrence of the corresponding outcome (the larger the piece, the larger the probability). In each lottery, the probabilities add up to 100%.

STEP 1

Assume that during a trip overseas, you and only you are kidnapped by a group of armed men. The leader tells you that you are going to be held in a closed room, alone, with no chance of escaping from it, being freed or communicating (with relatives or friends). You will be properly fed during the period of detention but will have no activity. You are told that your government will pay your usual earnings to your family throughout your detention.
The leader of the group offers you the opportunity to influence your situation by presenting you with a set of choices between two options that differ by the length of detention. You should fill in the length of detention in the blank space in such a way that you are absolutely indifferent between the two options. Indifference means that, given the length of detention

written, should you really have to choose between the two options, you would feel unable to decide between the two.

A flip of a coin will then determine which of the two options is to be applied to you (ZUW or ZOW). Then a random device based on the probabilities in the selected option will determine the length of your detention.

Remarks

You are allowed to choose non-integer periods (3.5 months or 6.7 months for instance).

Please check that you carefully turn one page after the other.

STEPS 2 AND 3

Assume again that you are kidnapped by a group of armed men. The leader tells you that you are going to be held in a closed room, alone, with no chance of escaping from it, being freed or communicating (with relatives or friends). You are told again that your government will pay your usual earnings to your family throughout your detention.

The leader of the group again offers you the opportunity to influence your situation by presenting you with a set of choices between two options that differ by the probabilities. You should now fill in the blank space in such a way that you are absolutely indifferent between the two options. Indifference means that, given the probability written, should you really have to choose between the two options, you would feel unable to decide between the two.

A flip of a coin will then determine which of the two options is to be applied to you (ZUW or ZOW). Then a random device based on the probabilities in the selected option will determine the length of your detention.

Remarks

You must choose probabilities between 0% and 100%. You may choose the probability you want (16.5% or 97.4% for instance) as long as it makes you indifferent between the two options.

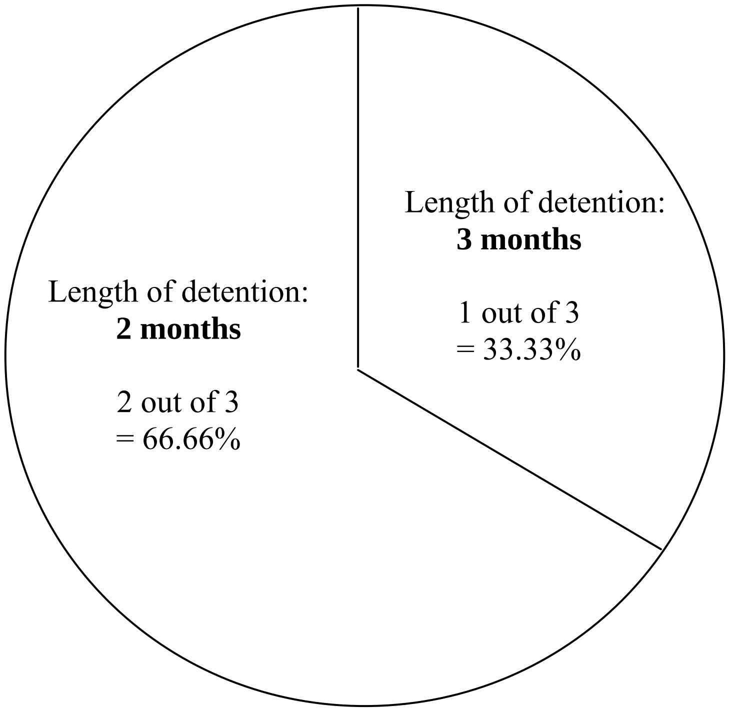
Please check that you carefully turn one page after the other.

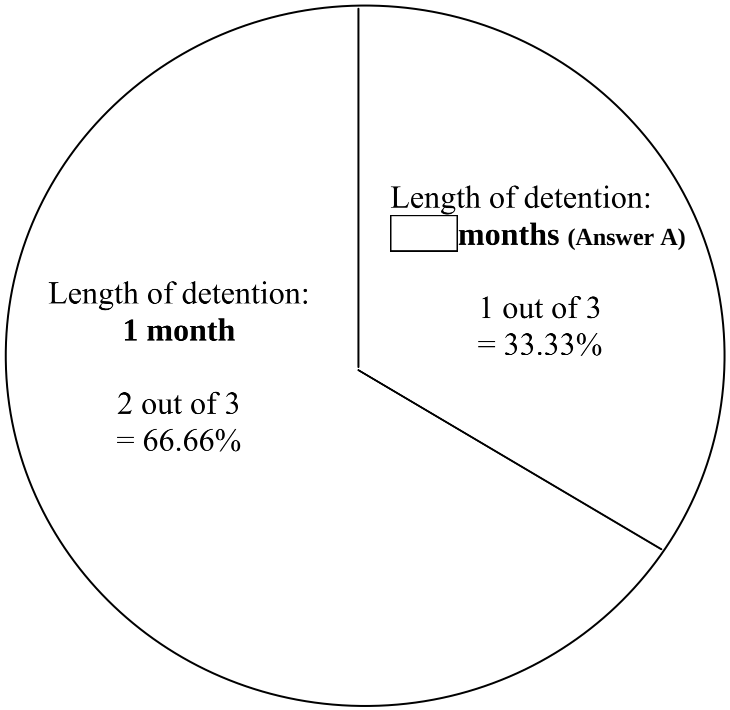
EXAMPLE OF THE TWO FIRST LOTTERIES PROPOSED IN STEP 1

ZUW OPTION

ZOW OPTION

Please write in the space below the length of time that makes you indifferent between option **ZUW** and option **ZOW**.



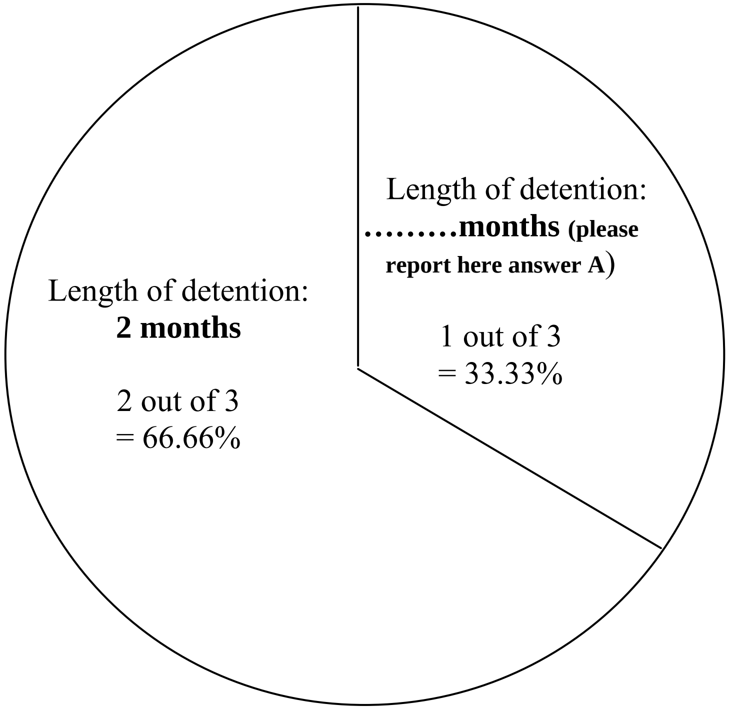


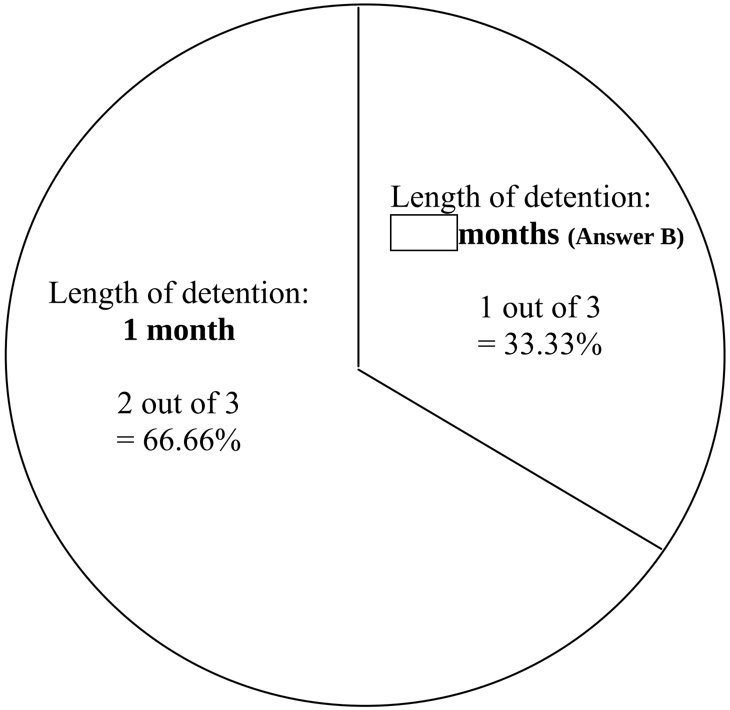
Remember that there are no right or wrong answers, only varying behaviors.
Take your time before answering.

ZUW OPTION

ZOW OPTION

Please write in the option **ZUW** your previous answer, then write in the space below the length of time that makes you indifferent between option **ZUW** and option **ZOW**.



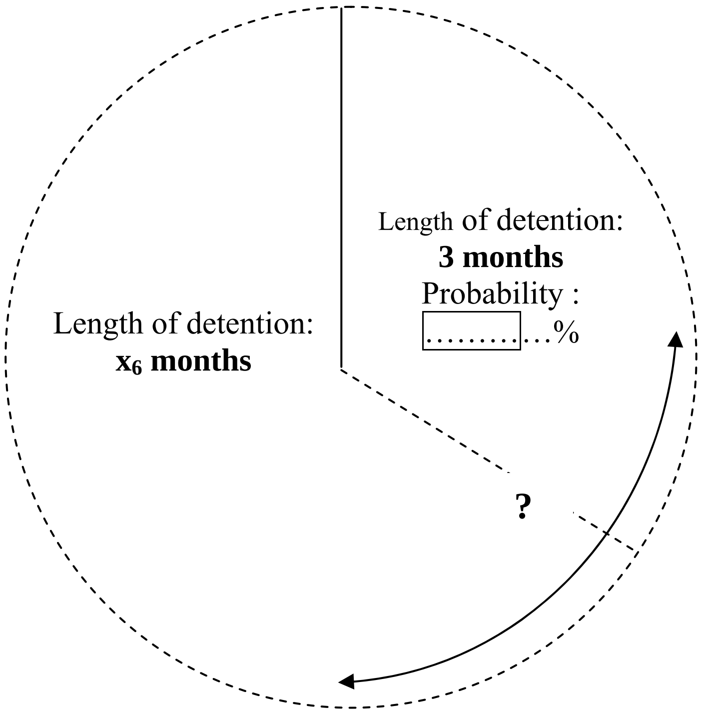


Remember that there are no right or wrong answers, only varying behaviors.
Take your time before answering

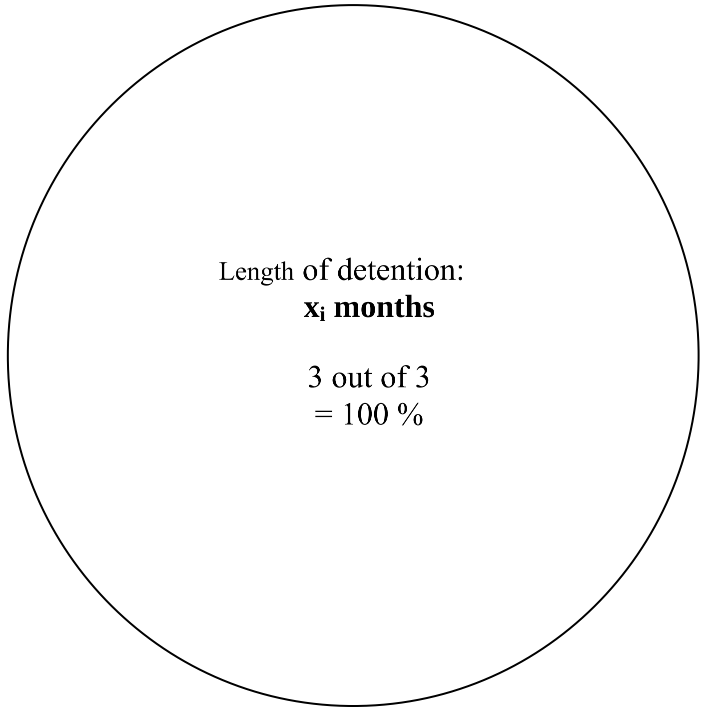
EXAMPLE OF THE LOTTERIES PROPOSED IN STEP 2 AND STEP 3

ZUW OPTION

Please write in the space below the probability that makes you indifferent between option **ZUW** and option **ZOW**.



ZOW OPTION



Remember that there are no right or wrong answers, only varying behaviors.
Take your time before answering

NB: **- In step 2, x_i (i=1 to 5) and x_6 are replaced by the corresponding values of the subject's standard outcome sequence.**
- In step 3, x_i is replaced by x_3 and x_6 is replaced by "40 years, i.e. 480 months".

APPENDIX 3: LOGARITHMIC COMMUNITY RISK SCALE

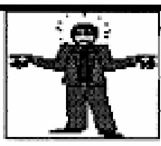
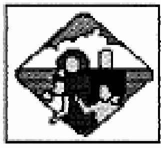
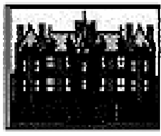
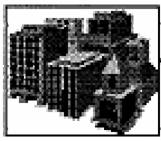
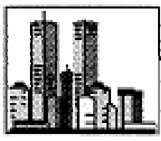
Risk magnitude (in percentage)		Expected about one event per	Examples (causes of death are expressed per year and for France)
1 in 1 (100%)		1 individual	Flip a coin = 1 in 2 (50%)
1 in 10 (10%)		10 ind. (family)	Draw six with a dice= 1 in 6 (16.66%) Draw two sixes with two dice = 1 in 36 (2.8%)
1 in 100 (1%)		100 ind. (small theatre)	Heart disease = 2.8 in 1000 (0.28%)
1 in 1 000 (0.1%)		1 000 ind. (village: Lamanon)	Die before 25 = 8 in 10 000 (0.08%)
1 in 10 000 (0.01%)		10 000 ind (town: Plan de Cuques)	Die in a car accident = 1 in 10 000 (0.01%)
1 in 100 000 (0.001%)		100 000 ind. (town: Aix en Provence)	Die in a fire = 4 in 1 000 000 (0.0004%)
1 in 1 000 000 (0.0001%)		1 million ind. (town: Marseille)	Die by lightening = 5 in 10 000 000 (0.000005%)
1 in 10 000 000 (0.00001%)		10 million ind. (Small country Belgium)	Match 6 out of 49 in National Lottery = 1 in 14 000 000 (0.000007%)
1 in 100 000 000 (0.000001%)		100 million ind. (large country Mexico)	A monkey writes « greqam » on a keyboard in one attempt = 3 in 1 000 000 000 (0.0000003%)
1 in 1 000 000 000 (0.0000001%)		1 billion ind. (Subcontinent : India)	Struck by a falling meteorite = 1 in 10 000 000 000 (0.00000001%)

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Table 1 Summary statistics on the easiness of answering the questions in the experiment

On a scale of 1 to 10, where '1' equates to 'very easy' and '10' equates to 'very difficult', circle the mark that best represents what you felt when you had to...	Mean[a] for the sample (n=42)	Mean[a] for excl. subj. (n=8)	P-value for testing the equ. of means
...answer the questions in the experiment	4.76 (1.68)	6.38 (2.33)	0.024
...choose a detention period in the lottery (Step 1)	4.90 (2.28)	7.88 (2.17)	0.001
...choose a probability in the lottery (Step 2)	5.40 (2.37)	7.13 (2.47)	0.067
...choose a probability in the last lottery (Step 3)	4.17 (2.84)	6.63 (3.02)	0.031
...change the response scale from a detention period (Step 1) to a probability (Steps 2 & 3)	4.43 (2.37)	6.75 (2.82)	0.017

[a]Standard deviation in brackets.

Table 2 Summary statistics on the usefulness of the aids provided to represent probability

On a scale of 1 to 10, where '1' equates to 'not useful at all' and '10' equates to 'very useful', circle the mark that best represents what you felt when you had to choose a probability	Mean	Std. Dev.	Range
The surfaces displayed on the blackboard	2.69	2.14	1-9
The sheets of paper laid out in the room	2.31	2.03	1-9
The probability scale expressed in "1 in X "	3.21	2.30	1-9
The probability scale expressed in percentage	4.24	2.98	1-9
The probability scale expressed with community analogies " 1 individual among ... "	3.45	2.70	1-10
The examples of various events involving probabilities of the same magnitude	3.45	2.65	1-10

Table 3 Classification of subjects in the non-catastrophic setting

Subject's elicited functions are compatible...	Frequency	Percentage
... with the EU framework		
... and $w(p) = p$ (strict EU)	5	11.91
... and $w(p) \neq p$ (RDEU)	36	85.71
... with a non-identical subjective treatment of probability	1	2.38
Total sample	42	100

Table 4 Detailed classification of subjects w.r.t. the EU framework

Utility function elicited in Step 1[a]	Probability function elicited in Step 2[a]	Value elicited in Step 3 shows...	Frequency[b]	Percentage
Concave (1)	Convex (1)	underweighting	1 (1)	2.38
Linear (35)	Concave (15)	underweighting	10 (4)	23.81
		EU-compatibility	5	11.90
	Identity (5)	EU-compatibility	5	11.90
	Convex (14)	underweighting	6 (4)	14.29
		EU-compatibility	5	11.90
		overweighting	3	7.14
	Mixed (1)	underweighting	1	2.38
Convex (6)	Concave (4)	underweighting	1 (1)	2.38
		EU-compatibility	1	2.38
		overweighting	2	4.76
	Convex (2)	EU-compatibility	1	2.38
		overweighting	1	2.38
Total			42 (10)	100

[a]Number of subjects in brackets.

[b]Number of subjects answering $p_{CAT} = 0$ are in brackets.

Table 5 Aggregate classification of subjects: non-catastrophic vs. catastrophic setting

	The subject's answer in Step 3 ...			
Subject's answers in Steps 1 & 2 are compatible...	is compatible with the EU framework	shows under-weighting	shows over-weighting	Total
...with the EU framework	17	18	6	41
...other[a]	0	1	0	1
Total	17 (40.48%)	19 (45.24%)	6 (14.28%)	42

[a]A non-linear subjective treatment of probability.

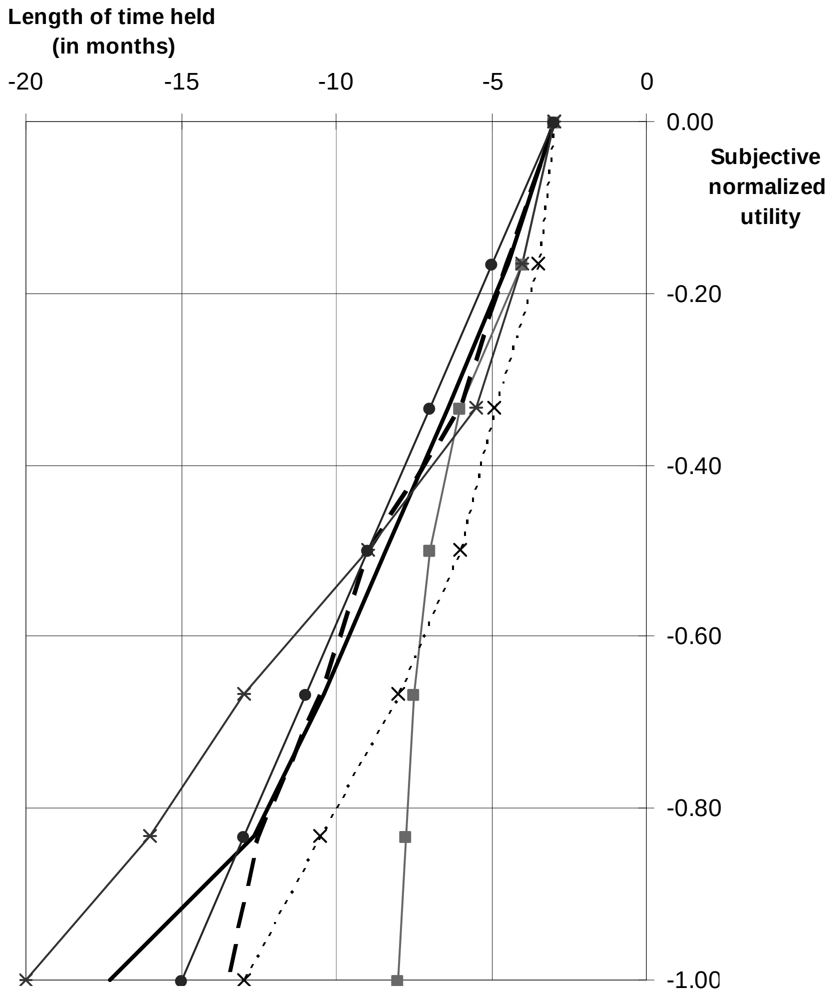

Figure 1 Utility functions elicited in Step 1. Examples showing convexity, concavity and linearity, average utility function in bold.

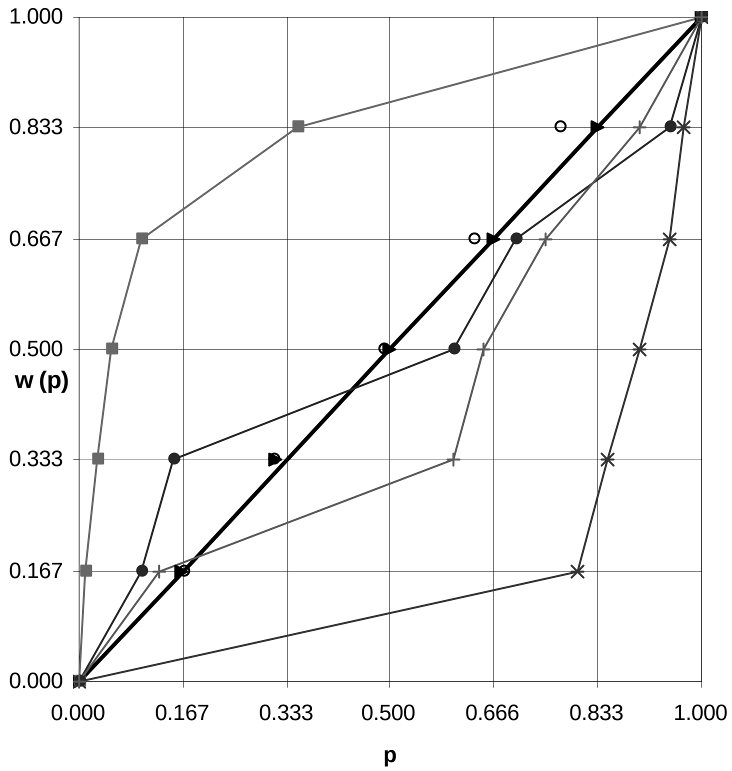

Figure 2 Probability functions elicited in Step 2. Linearity (dark line), the median probability weights (▲), the mean probability weights (○) and four subjects (other symbols) are represented.